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A STUDY OF THE MARKET OF TWO INTERCHANGEABLE GOODS FOR SUSTAINABILITY

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This paper examines the equilibrium of a market model of two interchangeable goods under conditions of critical and general cases. The influence of economic forces, such as prices, sales volumes, intensity of competition and model parameters, on the stability of the equilibrium is considered. The purpose of the work is to study the stability of the market equilibrium of two interchangeable goods depending on economic forces and model parameters. The research uses methods of mathematical modeling, differential equations and stability theory. A system of differential equations is introduced that describes the dynamics of the market for two interchangeable goods. It is shown that the stability of the market equilibrium of two interchangeable goods depends on the values of economic forces and model parameters. Equilibrium equations for a first-order model of two interchangeable goods in the general and critical cases are obtained. The stability of the equilibrium of a first-order model of two interchangeable goods is studied depending on the economic forces of sellers, merchants, the state and competition parameters. Examples of stable and unstable equilibria of a first-order model of two interchangeable goods are given. In particular, it was found that: if the prices for both goods are low enough, then the market equilibrium is stable; if the prices for both goods are high enough, then the market equilibrium is unstable; If the price of one good is low enough and the price of another good is high enough, then the market equilibrium can be either stable or unstable, depending on the values of other economic forces and model parameters. The results of the study can be used to predict the behavior of the market for two interchangeable goods depending on changes in economic conditions. They can also be used to develop recommendations for managing the market of two interchangeable goods in order to ensure its sustainability.

Key words: *market of two goods, interchangeable goods, equilibrium, stability, differential equations, price elasticity.*

Білоусова Т. П. Дослідження ринку двох взаємозамінних товарів на стійкість

У цій роботі досліджується рівновага моделі ринку двох взаємозамінних товарів в умовах критичного та загального випадків. Розглядається вплив економічних сил, таких як ціни, обсяги продажу, інтенсивність конкуренції та параметри моделі на стійкість рівноваги. Метою роботи є дослідження стійкості рівноваги ринку двох взаємозамінних товарів залежно від економічних сил та параметрів моделі. При дослідженні використовуються методи математичного моделювання, диференціальних рівнянь та теорії *стійкості. Запроваджується система диференціальних рівнянь, що визначає динаміку ринку двох взаємозамінних товарів. Показано, що стійкість рівноваги ринку двох взаємозамінних товарів залежить від значень економічних сил та параметрів моделі. Отримано рівняння рівноваги моделі першого порядку двох взаємозамінних товарів у загальному та критичному випадку. Досліджено стійкість рівноваги моделі першого порядку двох взаємозамінних товарів залежно від економічних сил продавців, торговців, держави та параметрів конкуренції. Наведено приклади стійких та нестійких рівноваг моделі першого порядку двох взаємозамінних товарів. Зокрема, було встановлено, що: якщо ціни на обидва товари є досить низькими, то рівновага ринку є стійкою; якщо ціни на обидва товари досить високі, то рівновага ринку є нестійкою; якщо ціна одного товару досить низька, а ціна іншого товару досить висока, то рівновага ринку може бути як стійкою, так і нестійкою, залежно від значень інших економічних сил і параметрів моделі. Сучасні математичні методи грають дедалі більшу роль в економіці. Результати дослідження можна використовувати для прогнозування поведінки ринку двох взаємозамінних товарів залежно від зміни економічних умов. А також можуть бути використані для розробки рекомендацій щодо управління ринком двох взаємозамінних товарів з метою забезпечення його стійкості.*

Ключові слова: *ринок двох товарів, взаємозамінні товари, рівновага, стійкість, диференціальні рівняння, цінова еластичність.*

Introduction. The study of market equilibrium, in particular the equilibrium of the first-order model of two interchangeable goods, has a rich history that goes back to the early stages of the development of economic theory. L. Walras is considered one of the founders of neoclassical economics. He developed a general equilibrium model that describes how prices and volumes are determined in all markets of the economy simultaneously. His work paved the way for further studies of equilibrium in the markets of individual goods [1]. D. Marshall expanded the Walras model, including the analysis of supply and demand, as well as the behavior of firms. He also investigated the effect of time on market equilibrium [2]. F. Knight investigated the dynamics of markets for two interchangeable goods, focusing on the role of uncertainty and expectations. He also analyzed the influence of information and institutional structures on equilibrium [3]. R. Solow developed a model of economic growth that describes how factors of production, such as capital and labor, affect the long-term growth of the economy. P. Krugman, explored various aspects of market equilibrium, including international trade theory, macroeconomics, and public sector economics. The study of market equilibrium remains a relevant topic. Increasing globalization has led to increased competition in world markets, which makes understanding the factors that determine market equilibrium even more important. Rapid technological progress leads to constant changes in the structure of the economy, which makes it necessary to constantly update market equilibrium models [4]. The economy faces many sources of uncertainty and risk, such as changes in commodity prices, political instability and natural disasters. Understanding how markets adapt to these changes is important for designing effective policies [5]. Researching the stability of substitute goods markets is an urgent task in the modern economy. In the conditions of globalization and the development of e-commerce, competition between goods is increasing, which leads to changes in their prices and sales volumes. Understanding the factors affecting the stability of markets of substitute goods allows us to predict their behavior and develop effective measures of state regulation [6]. The study of market equilibrium allows us to better understand how economies function, predict the behavior of markets, and develop effective policies.

Formulation of the problem. Let's consider a market model of two substitute goods [7]. Let us introduce the following notation: $p_j(t)$ is the price of a unit of the jth product at time t; p_j^0 is the equilibrium price of the j-th product; $q_j(t)$ is the number of units of the jth product sold at time t; q_j^0 – equilibrium number of units of the j-th product at price p_j^0; p_j^* – lower threshold value of the price of the j-th product associated with the seller's costs incurred; p_j^{**} is the upper ceiling value of the price of the j-th product, above which buyers refuse to purchase this product. Let $q_j(p)$ be a function of sales volumes of the jth goods at market prices $p=(p_1,p_2)$. The mathematical model of the market is presented in the form of a system of differential equations [8]

$$\dot{p}_1=-\frac{v_1\left(p_1-p_1^0\right)p_1'}{p_1-p_1^*}-\frac{d_1\left(p_1-p_1^0\right)p_1''}{p_1^{**}-p_1}-c_1\left(\left(p_1-p_1^0\right)-\left(p_2-p_2^0\right)\right)+\frac{r_1}{q_1^0}\left(p_1q_1(p)-p_1^0q_1^0\right),$$

$$\dot{p}_2=-\frac{v_2\left(p_2-p_2^0\right)p_2'}{p_2-p_2^*}-\frac{d_2\left(p_2-p_2^0\right)p_2''}{p_2^{**}-p_2}-c_2\left(\left(p_2-p_2^0\right)-\left(p_1-p_1^0\right)\right)+\frac{r_2}{q_2^0}\left(p_2q_2(p)-p_2^0q_2^0\right), \quad (1)$$

where $p_j^*<p_j<p_j^{**}$, $j=1,2,$ and, v_j, d_j, c_j, r_j are positive parameters of the model that characterize the intensity of economic forces.

Let us assume that the functions of sales volumes are specified linearly according to the formulas $q_j(p)=q_j^0\left(1-\frac{e_j}{p_j^0}\left(p_j-p_j^0\right)+\frac{e_{ji}}{p_i^0}\left(p_i-p_i^0\right)\right)$, $j,i=1,2, j\neq i,$

where e_j is the price elasticity of demand, and e_{ij} is the cross price elasticity [8]. These quantities are determined by the formulas:

$$e_j=-\frac{p_j^0}{q_j^0}\frac{\partial q_j\left(p^0\right)}{\partial p_j},\ e_{jk}=\frac{p_k^0}{q_j^0}\frac{\partial q_j\left(p^0\right)}{\partial p_k}, j,k=1,2, j\neq k, \text{ at point } p^0=\left(p_1^0,p_2^0\right).$$

Presenting main material. *1. Stability of balance.* Note that model (1) has an economic equilibrium $p_1=p_1^0$, $p_2=p_2^0$. To study the stability of this equilibrium, we will make a change of variables in system (1) $y_j=p_j-p_j^0, j=1,2$, having previously substituted expressions for functions of sales volumes, and use the following notation: $p_j'=p_j^0-p_j^*$ – seller's surplus price, $p_j''=p_j^{**}-p_j^0$ –consumer price surplus. Then we obtain a system of differential equations for the components of the vector $y=(y_1,y_2)$ in the following form:

$$\dot{y}_1=-\frac{v_1p_1'y_1}{y_1+p_1'}-\frac{d_1p_1''y_1}{p_1''-y_1}-c_1\left(y_1-y_2\right)+r_1\left(\left(1-e_1\right)y_1+e_{12}p_{12}^0y_2-\frac{e_1}{p_1^0}y_1^2+\frac{e_{12}}{p_2^0}y_1y_2\right),$$

$$\dot{y}_2=-\frac{v_2p_2'y_2}{y_2+p_2'}-\frac{d_2p_2''y_2}{p_2''-y_2}-c_2\left(y_2-y_1\right)+r_2\left(\left(1-e_2\right)y_2+e_{21}p_{21}^0y_1-\frac{e_2}{p_2^0}y_2^2+\frac{e_{21}}{p_1^0}y_1y_2\right), \quad (2)$$

where, $p_j'<y_j<p_j'', j=1,2$. Here, the economic equilibrium corresponds to the origin of coordinates $y_1=y_2=0$.

Let us select the linear and nonlinear parts of the equations of system (2) by writing

$$\dot{y}_1=-S_1y_1+R_1y_2+Y_1\left(y_1,y_2\right),\ \dot{y}_2=R_2y_1-S_2y_2+Y_2\left(y_1,y_2\right). \quad (3)$$

Here for the linear part it is assumed

$$S_j=v_j+d_j+c_j-r_j\left(1-e_j\right),\ R_j=c_j+r_jp_{ji}^0e_{ji},\ p_{ij}^0=p_i^0/p_j^0,\ j,i=1,2, \quad (4)$$

where S_j is the safety margin of the market for the jth product. The nonlinear part is determined by the expression

$$Y_j(y_1,y_2)=M_{1j}y_j^2+\frac{r_je_{jk}}{p_k^0}y_jy_k-H_{2j}y_j^3+o\left(\left\|y_j\right\|^3\right),\quad j,k=1,2,j\neq r,$$

where $o\left(\|y\|^3\right)$ means at $y\to 0$ a value of the order of smallness above the third, and it is assumed

$$H_{1j}=\frac{v_j}{p_j'}-\frac{d_j}{p_j''},\quad M_{1j}=H_{1j}-\frac{r_je_j}{p_j^0},\quad H_{2j}=\frac{v_j}{p_j'^2}+\frac{d_j}{p_j''^2}. \tag{5}$$

The first approximation model of system (3) takes the form of system

$$\begin{cases}\dot{y}_1=-S_1y_1+R_1y_2,\\ \dot{y}_2=R_2y_1-S_2y_2,\end{cases} \tag{6}$$

whose characteristic equation is given by the equality

$$\lambda^2-\lambda\left(S_1+S_2\right)+S_1S_2-R_1R_2=0. \tag{7}$$

According to the Rouse–Hurwitz criterion and the first approximation stability theorem, the conditions for the asymptotic stability of the equilibrium of system (3) are determined by the inequalities 1) $S_1>0,\ \ S_2>0$; 2) $S_1S_2-R_1R_2>0$.

These conditions suggest the following economic conclusions:

1. The safety margin of each of the products of market competitors must be strictly positive $\left(v_j+d_j+c_j-r_j\left(1-e_j\right)>0, j=1,2\right)$. In particular, it follows that the intensity of competitive forces plays a stabilizing role in the sustainable development of the market. The same positive role is played by the property of price elasticity of competing goods $\left(e_1\geq 1, e_2\geq 1\right)$.

2. The product of safety margins of both goods must be strictly separated from zero by an amount no less than the product of the intensity of the forces of competition, since $R_1R>c_1c_2$.

Further study of the stability of the economic equilibrium of the model is primarily related to critical cases when one of the roots of equation (7) has a zero real part. It is easy to verify that, taking into account the inequalities $R_j>0, j=1,2$, the model is only possible in the critical case of one zero root, for which the problem of stability is studied below. Consequently, in all other situations the equilibrium is unstable, which follows from Lyapunov's theorem on instability to the first approximation.

2. The critical case of one zero root. From equation (7) it is clear that such a case occurs when

$$S_1S_2=R_1R_2 \tag{8}$$

end

$$S_1>0, S_2>0. \tag{9}$$

To study the stability of economic equilibrium, we make a change of variables in system (6)

$$x_1=y_1,\quad x=a_1y_1+a_2y_2, \tag{10}$$

where the coefficients a_1 and a_2 are to be determined, and the non-degeneracy of the proposed transformation means that $a_2\neq 0$. We require that, as a result of such a replacement, $\dot{x} \equiv 0$. In this case, the coefficients a_j, $j = 1,2$, satisfy the following system of equations:

$$\begin{cases} S_1 a_1 - R_2 a_2 = 0 \\ R_1 a_1 - S_2 a_2 = 0. \end{cases} \tag{11}$$

Based on requirement (8), the determinant of such a system is equal to zero. Therefore, to find coefficients a_1 and a_2 we can take, for example, the first of the equations

$$S_1 a_1 - R_2 a_2 = 0 \tag{12}$$

Taking into account (8), solution (12), which meets the requirement $a_2 \neq 0$, exists only for $R_2 \neq 0$, $S_1 a_1 \neq 0$. Consequently, equation (12) has the family of solutions $a_1 = R_2 b / S_1$, $a_2 = b \neq 0$, where the parameter b will be refined further in order to simplify cumbersome expressions.

Thus, the desired change of variables (10) has the form

$$x = R_2 b y_1 / S_1 + b y_2, \quad x_1 = y_1, b \neq 0. \tag{13}$$

This implies feedback between the variables

$$y_1 = x_1, y_2 = x/b - R_2 x_1 / S_1, b \neq 0. \tag{14}$$

As a result, linear system (6) is transformed to the desired form

$$\begin{cases} \dot{x} = 0, \\ \dot{x}_1 = -(S_1 + S_2) x_1 + R_1 x/b, \quad b \neq 0. \end{cases}$$

Replacing variables (13) of the system under study (3) results in the nonlinear part of the second equation to the form $X(x, x_1) \equiv R_2 b Y_1(y_1, y_2) / S_1 + b Y_2(y_1, y_2)$, in which the variables (y_1, y_2) should be replaced by the formulas (14). The nonlinear part $X_1(x, x_1)$ is similarly transformed. As a result of such transformations we obtain the system

$$\begin{cases} \dot{x} = X(x, x_1), \\ \dot{x}_1 = -(S_1 + S_2) x_1 + R_1 x / b + X_1(x, x_1), \quad b \neq 0 \end{cases} \tag{15}$$

where, taking into account notation (5), we can write the nonlinear terms in the form

$$X(x, x_1) = \frac{M_{12}}{b} x^2 + \left(M_{12} \frac{S_2^2}{R_1^2} + M_{11} \frac{S_2}{R_1} - \left(\frac{S_2}{R_1} \frac{r_1 e_{12}}{p_2^0} + \frac{r_2 e_{21}}{p_1^0} \right) \frac{S_2}{R_1} \right) b x_1^2 +$$

$$+ \left(\frac{S_2}{R_1} \frac{r_1 e_{12}}{p_2^0} + \frac{r_2 e_{21}}{p_1^0} - 2 \frac{M_{12} S_2}{R_1} \right) x x_1 + \left(H_{22} \frac{S_2^3}{R_1^3} - \frac{S_2}{R_1} H_{21} \right) b x_1^3 - H_{22} \frac{1}{b^2} x^3 + 3 H_{22} \frac{1}{b} \frac{S_2}{R_1} x_1 x^2 -$$

$$-3 H_{22} \frac{S_2^2}{R_1^2} x_1^2 x + o\left(\left\| (x, x_1) \right\|^3 \right);$$

$$X_1(x, x_1) = \left(M_{11} - \frac{r_1 e_{12} S_2}{p_2^0 R_1} \right) x_1^2 + \frac{r_1 e_{12}}{b p_2^0} x_1 x - H_{21} x_1^3 + o\left(\left\| (x, x_1) \right\|^3 \right).$$

Putting $X^0(x) = X(x, 0)$ and $X_1^0(x) = X_1(x, 0)$, we are convinced that for the smallest powers in the variable x the following relations hold:

$$\min \deg\left(X_1^0(x) \right) = \min \deg\left(X_1^0(x, 0) \right) \geq \min \deg\left(X^0(x) \right) = \min \deg\left(X(x, 0) \right).$$

Therefore, we can move on to the next stage of studying the critical case. Bearing in mind that in equations (3) the variables were replaced according to formulas (13), we equate to zero the right-hand side of the non-critical (second) equation of system (15): $-(S_1+S_2)x_1+R_1x/b+X_1(x,x_1)=0.$

Let us find the solution $x_1=f(x)$ of this equation in implicit functions using the method of undetermined coefficients, putting $f(x)=\alpha x+\beta x^2+\gamma x^3+\ldots$. When finding the coefficients of a power series $f(x)$, one should take into account the fact that the minimum degree with which x_1 enters the expression $X(x,x_1)$ is degree two. Note that the parameter β after the replacement $x_1=f(x)$ is included in the nonlinear part $X(x,x_1)$, starting with the coefficients of the third power of x; subsequent parameters $-\gamma$, etc. are included in the expressions for $X(x,x_1)$, starting from higher powers of x, therefore, if we limit ourselves to only the coefficients of the first and second powers, such terms will not be included in the consideration. Equating the coefficients at the same powers, we find α and β from the system of equations

$$\begin{cases} (-S_1-S_2)\alpha+\dfrac{R_1}{b}=0, \\ \left(M_{11}-\dfrac{r_1e_{12}}{p_2^0}\dfrac{S_2}{R_1}\right)\alpha^2+\dfrac{r_1e_{12}}{p_2^0}\dfrac{1}{b}\alpha+(-S_1-S_2)\beta=0. \end{cases}$$

Let us set here $b=1/(S_1+S_2)^2$, then taking into account inequality (9) we obtain a solution to this system:

$$\alpha=(S_1+S_2)R_1\ ;\ \beta=(S_1+S_2)R_1\left(R_1p_2^0M_{11}+r_1e_{12}S_1\right)/p_2^0. \quad (16)$$

Thus, taking into account formulas (16), the right-hand side of the first equation of system (15) is transformed to the following form: $X(x,x_1)=Fx^2+Gx^3+o\left(\|x\|^3\right)$, where using the equality $R_2=S_1S_2/R_1$ (critical case condition) we will have

$$F=M_{12}S_1^2+M_{11}S_2R_1+\left(\frac{S_2}{R_1}\frac{r_1e_{12}}{p_2^0}+\frac{r_2e_{21}}{p_1^0}\right)S_1R_1\,;$$

$$G=\left(R_1^2M_{11}+R_1S_1\frac{r_1e_{12}}{p_2^0}\right)\left(2M_{11}S_2-2M_{12}R_2+\left(\frac{R_2}{S_1}\frac{r_1e_{12}}{p_2^0}+\frac{r_2e_{21}}{p_1^0}\right)(S_1-S_2)\right)-$$
$$-(S_1+S_2)\left(H_{22}S_1^3+S_2R_1^2H_{21}\right). \quad (17)$$

Note 1. If in our studies we use not the first of equalities (11), but the second, we will obtain formulas for F and G similar to formulas (17), but with the replacement of the indices 1 by 2 and 2 by 1, namely, we will have the expressions

$$F=M_{11}S_2^2+M_{12}S_1R_2+\left(\frac{S_1}{R_2}\frac{r_2e_{21}}{p_1^0}+\frac{r_1e_{12}}{p_2^0}\right)S_2R_2\,;$$

$$G=\left(R_2^2M_{12}+R_2S_2\frac{r_2e_{21}}{p_1^0}\right)\left(2M_{12}S_1-2M_{11}R_1+\left(\frac{R_1}{S_2}\frac{r_2e_{21}}{p_1^0}+\frac{r_1e_{12}}{p_2^0}\right)(S_2-S_1)\right)-$$
$$-(S_2+S_1)\left(H_{21}S_2^3+S_1R_2^2H_{22}\right). \quad (17^*)$$

Thus, the conducted studies of the model lead to the following statement.

Statement 1. The economic equilibrium $p_1 = p_1^0$, $p_2 = p_2^0$ of the market of two interchangeable goods is asymptotically stable if the margin of safety of each of the competing goods is strictly positive, i.e. $v_1 + d_1 + c_1 - r_1(1-e_1) > 0$, $v_2 + d_2 + c_2 - r_2(1-e_2) > 0$, and one of the following two conditions is met:

1) $\left(v_1 + d_1 + c_1 - r_1(1-e_1)\right)\left(v_2 + d_2 + c_2 - r_2(1-e_2)\right) > \left(c_1 + r_1\dfrac{p_1^0}{p_2^0}e_{12}\right)\left(c_2 + r_2\dfrac{p_2^0}{p_1^0}e_{21}\right)$, or

2) $\left(v_1 + d_1 + c_1 - r_1(1-e_1)\right)\left(v_2 + d_2 + c_2 - r_2(1-e_2)\right) = \left(c_1 + r_1\dfrac{p_1^0}{p_2^0}e_{12}\right)\left(c_2 + r_2\dfrac{p_2^0}{p_1^0}e_{21}\right)$,

$F = 0$, $G < 0$, where the values of F and G are determined by formulas (5), (17).

In all other cases, except $F = 0$, $G = 0$, the equilibrium is unstable.

In the case of $F = 0$, $G = 0$, further research is needed.

Note 2. It is easy to show that with appropriate values of the model parameters, the quantities F and G can take on values of different signs, including vanishing. Therefore, each of the options for the signs of the quantities F and G indicated in the statement can be implemented.

Note 3. In the critical case, in contrast to the study of the stability of economic equilibrium using the first approximation system, the conditions of stability or instability are cumbersome expressions that include all the parameters of the mathematical model. To explain these conditions based on the expressions F and G, let us consider the so-called absolutely symmetric case. We are talking about a situation where both competing products correspond to completely matching values of the model parameters, namely, $v = v_j$, $d = d_j$, $c = c_j$, $r = r_j$, $p = p_j^0$, $p_j' = p'$, $p_j'' = p''$, $j = 1,2$, $e = e_1 = e_2, e_{12} = e_{21}$. In this case we have $R = S$ and then $F = 2S^2\left(\dfrac{v}{p'} - \dfrac{d}{p''} - \dfrac{re}{p^0} + \dfrac{re_{12}}{p^0}\right)$. Consequently, the conditions that $F = 0$ or $F \neq 0$ are respectively equivalent to the relations: $\dfrac{v}{p'} - \dfrac{d}{p''} = \dfrac{r(e - e_{12})}{p^0}$ or $\dfrac{v}{p'} - \dfrac{d}{p''} \neq \dfrac{r(e - e_{12})}{p^0}$.

From an economic point of view, the first of these equalities means a certain parity between producers, consumers and the state, taking into account the properties of price elasticity of demand and cross-price elasticity. Therefore, it follows from the theorem that if this parity is violated, i.e. $F \neq 0$, the model is unstable. In the case of $F = 0$, the expression for G is simplified and takes the form $G = -4S^4H_2 < 0$. Therefore, we obtain the asymptotic stability of economic equilibrium.

3. Constant sales volume. For a special case with a constant sales volume $q_j(p) = q_j^0$, $j = 1,2$, the model is described by a system of differential equations

$$\begin{cases} \dot{p}_1 = -\dfrac{v_1\left(p_1 - p_1^0\right)p_1'}{p_1 - p_1^*} - \dfrac{d_1\left(p_1 - p_1^0\right)p_1''}{p_1^{**} - p_1} + r_1\left(p_1 - p_1^0\right) - c_1\left(\left(p_1 - p_1^0\right) - \left(p_2 - p_2^0\right)\right), \\ \dot{p}_2 = -\dfrac{v_2\left(p_2 - p_2^0\right)p_2'}{p_2 - p_2^*} - \dfrac{d_2\left(p_2 - p_2^0\right)p_2''}{p_2^{**} - p_2} + r_2\left(p_2 - p_2^0\right) - c_2\left(\left(p_2 - p_2^0\right) - \left(p_1 - p_1^0\right)\right). \end{cases}$$

The conditions for asymptotic stability in the first approximation for such a system are given by the inequalities 1) $S_1 > 0, S_2 > 0$; 2) $S_1 S_2 - c_1 c_2 > 0$, where the safety margin of competing goods is determined by the formula $S_j = v_j + d_j + c_j - r_j$, $j = 1,2$. For the critical case, when $S_1 S_2 = c_1 c_2$ formulas (17) respectively take the form $F = H_{12} S_1^2 + H_{11} S_2 R_1$; $G = 2H_{11} R_1^2 \left(H_{11} S_2 - H_{12} R_2 \right) - \left(S_1 + S_2 \right) \left(H_{22} S_1^3 + H_{21} S_2 R_1^2 \right)$.

Formulas (17*) are transformed in a similar way. In this case, the conclusions of an economic nature noted in Note *3* are formally obtained under the condition that price elasticity ($e = 0$) and cross-price elasticity ($e_{12} = 0$) are equal to zero. At $F = 0$, i.e. $H_{12} = -H_{11} S_2 c_1 / S_1^2$, and $S_1 S_2 = c_1 c_2$ (the condition for the critical case of one zero root), the expression G is transformed to the form

$$G = \left(S_2 + S_1 \right) \left(2H_{11}^2 c_1^2 S_2 - H_{22} S_1^4 - H_{21} S_2 S_1 c_1^2 \right) / S_1.$$

Therefore, here the conditions for asymptotic stability and instability, respectively, take the form of inequalities $c_1^2 S_2 \left(2H_{11}^2 - H_{21} S_1 \right) < H_{22} S_1^4$ end $c_1^2 S_2 \left(2H_{11}^2 - H_{21} S_1 \right) > H_{22} S_1^4$.

3.1. The case of identical parameters of competitors. If, as above, we assume that the goods are competitors are equal, i.e. their parameters completely coincide, then

$F = SH_1 \left(S + c \right)$. Provided that $F = 0$ (i.e. $H_1 = 0$) and $S = R = c$ (the condition for the critical case of one zero root), the expression G is transformed to the form $G = -4c^4 H_2$. Therefore, the following statement follows from Statement 1.

Statement 2. Let the following conditions hold for a market of two interchangeable goods:

1) $v = v_j$, $d = d_j$, $c = c_j$, $r = r_j$, $p = p_j^0$, $p_j' = p'$, $p_j'' = p''$, $j = 1,2$, $e = e_1 = e_2$, $e_{12} = e_{21}$;

2) sales volumes are constant $q_j \left(p \right) = q_j^0$, $j = 1,2$.

Then the economic equilibrium $p_1 = p_1^0$, $p_2 = p_2^0$ is asymptotically stable if one of the following two conditions is satisfied: 1) $v + d > r$ or 2) $v + d = r$, $\dfrac{v}{p'} = \dfrac{d}{p''}$.

In all other cases, the equilibrium is unstable.

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