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## APPLICATION OF SET THEORY FOR THE MATHEMATICAL JUSTIFICATION OF DEVELOPING AN IOT SYSTEM FOR AUTOMATED SOIL MOISTURE MONITORING

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The article explores the application of set theory for the mathematical substantiation of an IoT-based system designed for automated soil moisture monitoring. The innovative approach integrates the concepts of the Internet of Things with modern automation methods to ensure efficient management of agricultural resources. The theoretical framework of the study focuses on developing a mathematical model that employs set theory principles to formalize the relationships between sensors, monitoring parameters, external conditions, and data processing algorithms. Special attention is given to the development of algorithms for collecting, analyzing, and processing data from soil moisture sensors, enabling continuous soil condition monitoring. These algorithms utilize set-theoretic operations to merge data from various sensors and filter out noise. Based on the proposed models, the article presents the architecture of an IoT system that includes hardware components such as sensors and controllers, network data transmission protocols, a cloud environment for data storage and analysis, and an interactive user interface. A detailed analysis of the practical aspects of system implementation is provided, covering sensor specifications, energy consumption, network stability, interoperability, and integration with external weather prediction modules. Particular emphasis is placed on data security during transmission and system resilience to failures. The expected outcomes of implementing the system include optimized irrigation processes, increased crop yields, reduced resource consumption, and minimized environmental impact. The system facilitates timely responses to changes in soil moisture levels, enabling informed resource management decisions. Additionally, the article examines the potential for scaling the IoT system to other agricultural applications, such as monitoring soil temperature, light levels, and acidity indicators. The findings validate the feasibility of using set theory to construct formal models that enhance monitoring accuracy, accelerate data analysis, and improve management efficiency in the agricultural sector.

**Key words:** information system, IoT system, mathematical modeling, set theory, soil moisture control, soil moisture monitoring.

**Зюзиун В. І., Стародубець В. А. Застосування теорії множин для математичного обґрунтування розробки проєкту створення IoT-системи для автоматичного моніторингу рівня вологості ґрунтів**

У статті розглядається застосування теорії множин для математичного обґрунтування розробки проєкту створення IoT-системи для автоматичного моніторингу рівня вологості ґрунтів. Інноваційний підхід базується на інтеграції концепцій Інтернету речей із сучасними методами автоматизації для забезпечення ефективного управління сільськогосподарськими ресурсами. Теоретична частина статті зосереджена на побудові математичної моделі, яка використовує принципи теорії множин для формалізації відносин між сенсорами, параметрами моніторингу, зовнішніми умовами та алгоритмами обробки даних. Особливу увагу приділено розробці алгоритмів збору, аналізу та обробки даних із сенсорів вологості, які забезпечують безперервний контроль стану ґрунту. Ці алгоритми базуються на використанні теоретико-множинних операцій для об'єднання даних із різних сенсорів і фільтрації шуму. На основі запропонованих моделей представлено архітектуру IoT-системи, що включає компоненти апаратного забезпечення, такі

як сенсори та контролери, мережеві протоколи передачі даних, хмарне середовище для зберігання та аналізу інформації, а також інтерактивний інтерфейс для користувача. Проведено детальний аналіз практичних аспектів впровадження системи, включаючи технічні характеристики сенсорів, енергоспоживання, стабільність зв'язку, інтероперабельність мережі та інтеграцію із зовнішніми модулями прогнозування погодних умов. Значна увага приділяється питанням безпеки даних, що передаються, та стійкості системи до відмов. Очікувані результати застосування системи включають оптимізацію поливу, підвищення врожайності, зменшення витрат на ресурси та зниження впливу на навколишнє середовище. Система дозволяє вчасно реагувати на зміни рівня вологості та приймати обґрунтовані рішення щодо управління ресурсами.

Окрім цього, стаття розглядає перспективи масштабування IoT-системи для застосування в інших агротехнологіях, таких як контроль температури ґрунту, рівня освітленості та показників кислотності. Отримані результати підтверджують доцільність використання теорії множин для побудови формальних моделей, що підвищують точність моніторингу, швидкість аналізу даних та ефективність управління у сфері агро-сектору.

**Ключові слова:** інформаційна система, IoT-система, математичне моделювання, теорія множин, контроль вологості ґрунтів, моніторинг вологості ґрунтів.

**Introduction.** Modern agriculture faces a range of challenges, including the growing demand for efficient use of natural resources, the impact of climate change, and the need to optimize production processes to increase crop yields. One of the most pressing issues is the rational management of water resources, which directly affects soil health and agricultural productivity. Traditional methods of monitoring soil moisture levels are often insufficiently accurate, labor-intensive, or unable to provide real-time data.

The Internet of Things (IoT) offers innovative approaches to automating and managing agricultural processes. Utilizing IoT systems for monitoring soil moisture levels opens up opportunities for precise data collection, analysis, and processing, enabling prompt and well-informed decision-making. At the same time, designing such systems requires a robust theoretical foundation that accounts for the complexity of interactions among various components, the influence of external factors, and the needs of stakeholders.

One of the promising approaches to addressing this challenge is the application of set theory to develop a mathematical model that formalizes the system's key parameters. This approach enables a clear structuring of relationships among material, human, software, and informational resources, while also accounting for the impact of the external environment and the project's parameter requirements.

**The aim of the research.** The development of a mathematical foundation for designing an IoT system for automated soil moisture monitoring, based on the principles of set theory, aims to improve monitoring accuracy, optimize the management of agricultural resources, and minimize environmental impact.

**Analysis of recent research and publications.** In reference [1], key technological solutions for smart farming are presented, covering the use of IoT for monitoring soil moisture, light, and temperature. Special attention is given to critical challenges, such as ensuring reliable connectivity, managing energy consumption, and securing data. IoT technologies greatly enhance the efficiency of agricultural management by enabling real-time monitoring and optimizing resource usage.

In publication [2], the advantages of real-time monitoring of agricultural processes using IoT systems are examined. It provides a detailed explanation of how IoT helps optimize the use of water resources, monitor soil conditions, and improve crop yields. A key aspect highlighted is the integration of technologies that automate monitoring, ensuring farmers have access to accurate and timely data.

In study [3], the paper explores the key devices, platforms, network protocols, and data processing technologies relevant to agriculture.

In publication [4], the ten most significant IoT applications in 2022 are presented, with a focus on smart cities, logistics, and agriculture. It highlights the rapid advancement of IoT technologies and their growing impact across various industries, emphasizing the importance of integrating these systems for more effective management.

Publication [5] discusses the role of IoT in supporting sustainable agriculture, which helps reduce the environmental impact of agricultural practices. IoT systems enable efficient monitoring of crop growth conditions and optimize water and fertilizer consumption, lowering the ecological footprint. Additionally, these technologies contribute to increased yields and reduced resource costs, which is vital for the sustainable development of the agricultural sector.

In study [6], methods for optimizing resource management in agriculture through IoT integration are explored, while paper [7] emphasizes the role of big data analytics in enhancing the performance of IoT systems.

**Presentation of the main research material.** The proposed IoT system for automated soil moisture monitoring will consist of several key components that ensure its functionality and interaction with the external environment. This system is designed to maintain optimal conditions for the growth of agricultural crops through the use of modern monitoring and irrigation management technologies.

*The broader framework of the system* includes external factors that influence its operation and define its needs and requirements. Climate conditions necessitate continuous soil moisture monitoring, while environmental organizations may offer guidelines for the sustainable use of resources. Research institutions play a role in implementing new technologies and methodologies. Ultimately, the consumers of the products benefit from the system, as they receive high-quality produce.

*The external environment* encompasses all external influences on the system, such as climatic conditions and regulatory requirements. These factors may vary depending on geographical location and other conditions.

*The office area* includes rooms used for administrative and managerial functions, while the equipment room contains the necessary hardware to support the system's operation. The break room provides a comfortable environment for the staff, and the restrooms ensure basic sanitary conditions.

*Equipment.* Soil moisture sensors (FC-28) are installed directly in the soil to continuously monitor its condition. The ESP8266 microcontroller processes data from the sensors and transmits it to the server. The server collects all the data, enabling analysis, storage, and irrigation management. Computers are used for working with the database and system interface.

*Team members.* The project documentation manager is responsible for organizing and maintaining all project documentation. The agricultural consultant provides specialized knowledge to optimize processes. The project manager coordinates all stages of system implementation and operation. The accountant manages financial operations, while the technical team ensures technical support and equipment maintenance.

*User interface.* The user interface enables operators to seamlessly interact with the system, facilitating monitoring and control. A remote irrigation activation button streamlines the irrigation process by automating it based on real-time sensor data. A detailed field map provides a visual representation of soil moisture distribution across different areas, allowing for targeted decision-making. The system's database securely stores all collected data, supporting comprehensive analysis and informed decision-making.

*External Elements.* The IoT system collaborates with a network of stakeholders, including clients, technology providers, financial institutions, environmental

organizations, and research institutions. These external entities supply essential resources, information, and expertise, ensuring the system's efficient functionality and long-term success.

Thus, the IoT system for automated soil moisture monitoring represents a comprehensive solution that integrates various components and subsystems for the effective management of agricultural processes.

The conceptual model of this IoT system is shown in Fig. 1.

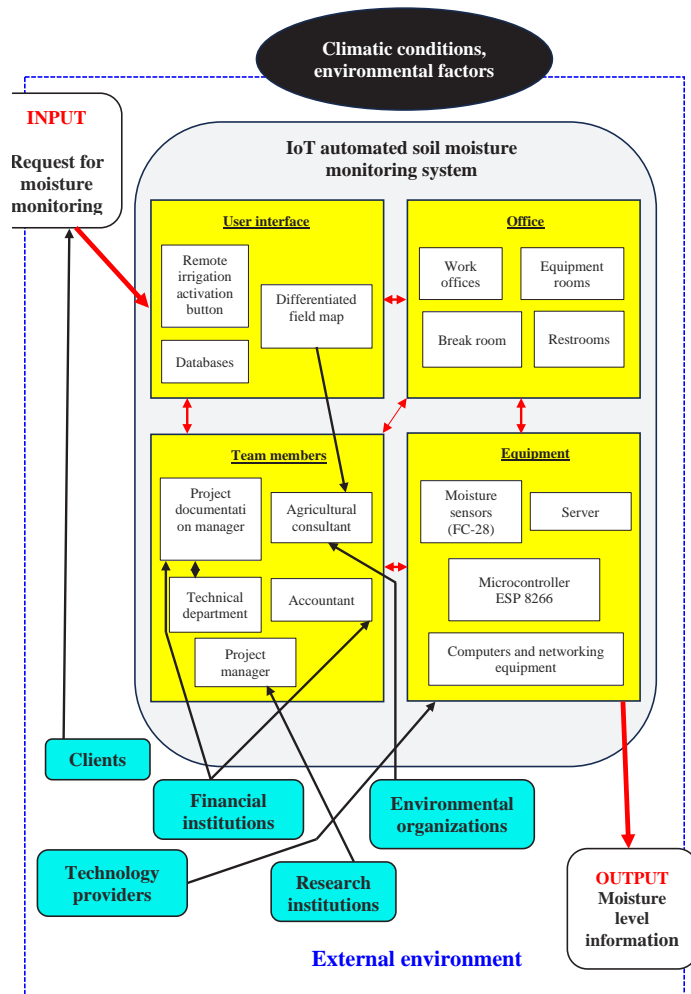


Fig. 1. Conceptual model of the IoT system

Set theory is a versatile mathematical tool that facilitates the creation of formal models for complex systems comprising numerous components with diverse properties and interconnections. Its application in the development of an IoT system for automated soil moisture monitoring is particularly advantageous, as it enables the design of a clear and adaptive system architecture capable of responding to dynamic changes in external conditions and internal parameters.

The significance of this approach is substantiated by various studies. For instance, the authors in [8] examines the collaboration between Industry 4.0 and IoT, demonstrating their potential to revolutionize industries through intelligent, automated systems that boost productivity and cut costs. It highlights IoT's applications in agriculture, healthcare, and logistics, offering solutions for precision farming, smart greenhouses, and enhancing transparency in supply chain operations. In [9], explores the tools and equipment used in agricultural IoT applications and the challenges of integrating these technologies with traditional farming practices. It highlights the benefits of IoT and cloud computing for enhancing farming processes, from sowing to packaging and transportation. Study [10] outlines data filtering methods based on set-theoretic operations, which can be adapted for sensor networks. Moreover, studies [11–12] demonstrate the use of set theory and fuzzy sets to construct mathematical models addressing environmental and ecological challenges.

IoT systems designed for monitoring soil moisture levels consist of numerous interconnected components, including sensors, software, hardware, network protocols, and external factors such as climatic conditions. Set theory offers a formal framework to model these relationships, providing a robust mathematical foundation for defining the hierarchy of system elements and their interactions. This framework enhances comprehension of the system's functional structure, streamlines architectural design, and ensures a modular approach.

In addition, the tools of set theory play a vital role in organizing and processing large datasets. IoT systems generate vast volumes of data that require efficient structuring and analysis. By applying set-theoretic operations like union, intersection, and difference, data can be organized, duplicates removed, irrelevant information filtered out, and critical insights highlighted. This method greatly increases the speed and precision of data analysis, which is essential for promptly identifying changes in soil conditions and making well-informed decisions.

A key advantage of using this mathematical tool is its support for optimizing project resource management. When designing an IoT system, various resources must be considered, such as material resources (sensors, controllers, network equipment), human resources, software, and information. Set theory provides an efficient way to model these resources as sets with defined properties and interrelationships. This approach allows for balancing their usage, preventing excessive expenditure, and ensuring the system's robustness to changes or unforeseen failures.

Additionally, set theory contributes to adaptability to external demands. The real-world operating environment of an IoT system is dynamic and complex, including factors like weather conditions, soil types, seasonality, and stakeholder requirements. Using set theory, these factors can be represented as separate sets that interact with the system. This enables the creation of adaptive algorithms that automatically account for changes in the environment, ensuring the system operates reliably and efficiently even in unpredictable circumstances.

Therefore, the application of set theory provides a structured and systematic approach to designing an IoT system for soil moisture monitoring. This approach not only formalizes the system's structure but also enhances its performance by ensuring accuracy, flexibility, and adaptability to real-world conditions.

The mathematical model will primarily focus on evaluating the quality, risks, and effectiveness of the project, considering all the required resources and parameters. Set theory will be employed to construct the mathematical model.

The proposed model, in general terms, will have the following expression:

$$M = \{G^M, P^H, R^P, I\}, \quad (1)$$

$G$  is the parameter that characterizes the set of material resources involved in the project, which is defined as:

$$G = \{g_1^M, g_2^M, g_3^M\}, \quad (2)$$

where  $g_1^M$  – FC28 sensor;  $g_2^M$  – ESP8266 microcontroller;  $g_3$  – irrigation system.

$P^H$  is the parameter that characterizes the set of human resources participating in the project, which is defined as:

$$P^H = \{p_1^H, p_2^H, p_3^H, p_4^H, p_5^H, p_6^H, p_7^H, p_8^H, p_9^H\}, \quad (3)$$

where  $p_1^H$  – project manager;  $p_2^H$  – system architect;  $p_3^H$  – designer;  $p_4^H$  – tester;  $p_5^H$  – developer;  $p_6^H$  – analyst;  $p_7^H$  – agricultural consultant;  $p_8^H$  – documentation manager;  $p_9^H$  – accountant.

$R^P$  is the parameter that characterizes the set of software resources used in the project, which is defined as:

$$R^P = \{r_1^P, r_2^P, r_3^P\}, \quad (4)$$

where  $r_1^P$  – Python programming language;  $r_2^P$  – SQL database;  $r_3^P$  – web server.

$I$  is the parameter that characterizes the set of informational resources in the project, which is defined as:

$$I = \{i_1^I, i_2^I, i_3^I, i_4^I\}, \quad (5)$$

where  $i_1^I$  – data from moisture sensors;  $i_2^I$  – data on types of grain crops;  $i_3^I$  – GIS (Geographic Information System);  $i_4^I$  – technical documentation and guidelines.

Stakeholders, who have a direct impact not only on the functionality of the future product of the project but also on the success of the entire project, significantly influence the success of the IT project.

The project stakeholders can be represented as a set of parameters:

$$O = \{O^L, O^D\}, \quad (6)$$

where  $O^L$  – parameter that characterizes the stakeholders (long-term project environment);  $O^D$  – parameter that characterizes the set of project participants (close project environment).

Let's examine the parameter  $O^L$  in more detail as the set of subsets that compose it:

$$O^L = \{o_1^L, o_2^L, o_3^L, o_4^L, o_5^L\}, \quad (7)$$

where  $o_1^L$  – farmers and agricultural enterprises;  $o_2^L$  – manufacturers and distributors of modern agricultural technologies;  $o_3^L$  – government agencies and regulators;  $o_4^L$  – environmental organizations;  $o_5^L$  – research and educational institutions.

Let's examine the parameter  $O^D$  in more detail as the set of subsets that compose it:

$$O^D = \{o_1^D, o_2^D\}, \quad (8)$$

where  $o_1^D$  – project team;  $o_2^D$  – project sponsors.

For IT projects, the creation of complex IT products is characterized by a service component that requires strategic services, development services, transitions, operations, and continuous improvements. The set of relevant parameters can be represented as a combination of:

$$S = \{S^I, S^B, S^{US}\}, \quad (9)$$

where  $S^I$  – parameter that characterizes the set of services supporting the IT infrastructure;  $S^B$  – parameter that characterizes the set of services supporting the business application;  $S^{US}$  – parameter that characterizes the set of services providing user support.

Let's examine the parameter  $S^I$  in more detail as the set of subsets that compose it:

$$S^I = \{s_1^I, s_2^I, s_3^I\}, \quad (10)$$

where  $s_1^I$  – monitoring and management of the moisture sensor system;  $s_2^I$  – storage and analysis of sensor data;  $s_3^I$  – web server support.

Let's examine the parameter  $S^B$  in more detail as the set of subsets that compose it:

$$S^B = \{s_1^B, s_2^B, s_3^B\}, \quad (11)$$

where,  $s_1^B$  – development and support of software for data processing;  $s_2^B$  – development and support of the user interface;  $s_3^B$  – development and support of the web server.

Let's examine the parameter  $S^{US}$  in more detail as the set of subsets that compose it:

$$S^{US} = \{s_1^{US}, s_2^{US}, s_3^{US}\}, \quad (12)$$

where  $s_1^{US}$  – technical support for system users (helpline);  $s_2^{US}$  – usage consultations;  $s_3^{US}$  – introductory course on operation.

As mentioned earlier, the use of technologies for creating the IT product was discussed. Additionally, this group also includes technologies for development, management, testing, operation, and maintenance of the IT project product. The set of relevant parameters  $W$  can be represented as a combination of:

$$W = \{W^C, W^M, W^I\}, \quad (13)$$

where  $W^C$  – parameter that characterizes the set of technologies for product development and testing;  $W^M$  – parameter that characterizes the set of project management technologies;  $W^I$  – parameter that characterizes the set of technologies for implementing and supporting the project product.

Let's examine the parameter  $W^C$  in more detail as the set of subsets that compose it:

$$W^C = \{w_1^C, w_2^C, w_3^C, w_4^C\}, \quad (14)$$

where  $w_1^C$  – python;  $w_2^C$  – SQL;  $w_3^C$  – UNIT testing for code correctness verification;  $w_4^C$  – frameworks for rapid server deployment.

Let's examine the parameter  $W^M$  in more detail as the set of subsets that compose it:

$$W^M = \{w_1^M, w_2^M\}, \quad (15)$$

where  $w_1^M$  – Project Management Software for planning and controlling project work;  $w_2^M$  – AGILE technologies for efficient project management.

Let's examine the parameter  $W^I$  in more detail as the set of subsets that compose it:

$$W^I = \{w_1^I, w_2^I, w_3^I, w_4^I, w_5^I, w_6^I\}, \quad (16)$$

where  $w_1^I$  – python;  $w_2^I$  – SQL;  $w_3^I$  – web server;  $w_4^I$  – sensors;  $w_5^I$  – microcontrollers;  $w_6^I$  – irrigation system.

The components of the IT product configuration are related to the definition of elements, their parameters, and their relationship to the developed information system. The same applies to the elements of the project and the project environment. The set of relevant parameters can be represented as a combination of:

$$K = \{K^P, K^S, K^E, K^{DP}, K^{DS}\}, \quad (17)$$

where  $K^P$  – the set of project parameters;  $K^S$  – the set of project product parameters;  $K^E$  – the set of parameters of the project's external environment;  $K^{DP}$  – the set of project requirements;  $K^{DS}$  – the set of product requirements.

Let's examine the parameter  $K^P$  in more detail as the set of subsets that compose it:

$$K^P = \{k_1^P, k_2^P, k_3^P, k_4^P\}, \quad (18)$$

where  $k_1^P$  – funding volume;  $k_2^P$  – project duration;  $k_3^P$  – project team composition;  $k_4^P$  – resources.

Let's examine the parameter  $K^S$  in more detail as the set of subsets that compose it:

$$K^S = \{k_1^S, k_2^S, k_3^S, k_4^S\}, \quad (19)$$

where  $k_1^S$  – accuracy of soil moisture measurement;  $k_2^S$  – system scalability;  $k_3^S$  – energy efficiency;  $k_4^S$  – user-friendliness for end-users.

Let's examine the parameter  $K^E$  in more detail as the set of subsets that compose it:

$$K^E = \{k_1^E, k_2^E, k_3^E, k_4^E\}, \quad (20)$$

where  $k_1^E$  – climatic conditions of the project implementation region;  $k_2^E$  – availability of infrastructure for sensor installation;  $k_3^E$  – potential risks and threats to the project;  $k_4^E$  – socio-economic context.

Let's examine the parameter  $K^{DP}$  in more detail as the set of subsets that compose it:

$$K^{DR} = \{k_1^{DP}, k_2^{DP}, k_3^{DP}, k_4^{DP}, k_5^{DP}\}, \quad (21)$$

where  $k_1^{DP}$  – defining the project implementation strategy, considering potential risks and taking steps to mitigate them;  $k_2^{DP}$  – developing the budget and planning the use of financial resources, including creating control and reporting mechanisms;  $k_3^{DP}$  – forming and managing the project team, defining roles and responsibilities of participants;  $k_4^{DP}$  – establishing a monitoring system for the project's progress and preparing reports to assess achievements and identify potential issues;  $k_5^{DP}$  – developing a project completion plan and transitioning to its operation or further development.

Let's examine the parameter  $K^{DS}$  in more detail as the set of subsets that compose it:

$$K^{DS} = \{k_1^{DS}, k_2^{DS}, k_3^{DS}, k_4^{DS}, k_5^{DS}, k_6^{DS}\}, \quad (22)$$

where  $k_1^{DS}$  – the system must enable real-time monitoring of soil moisture to respond quickly to changes and take necessary actions;  $k_2^{DS}$  – energy efficiency;  $k_3^{DS}$  – the system must have a user-friendly interface and be easy to use for farmers and other stakeholders. It should also be easily integrated with existing agricultural management systems;  $k_4^{DS}$  – the system must provide accurate and reliable data collection on soil moisture for further analysis;  $k_5^{DS}$  – the system must provide tools for data analysis and report generation, allowing users to obtain useful information for decision-making;  $k_6^{DS}$  – the system must ensure a high level of data security to protect the confidentiality and integrity of the information.

The parameters of the project product are presented as a set of sets:

$$L = \{L^S, L^P\}, \quad (23)$$

where  $L^S$  – the set of throughput values of the product components;  $L^P$  – the total number of throughput capabilities of all product components.

Let's examine the parameter  $L^S$  in more detail as the set of subsets that compose it:

$$L^S = \{l_1^S, l_2^S\}, \quad (24)$$

where  $l_1^S$  – data collection system: 1000 data/sec (500 operations/min);  $l_2^S$  – data analysis system: 100 data/sec (500 operations/min).

Let's examine the parameter  $L^P$  in more detail as the set of subsets that compose it:

$$L^P = \{l_1^P, l_2^P\}, \quad (25)$$

where  $l_1^P$  – data collection;  $l_2^P$  – data analysis.

The parameters of the project product are presented as  $A$  set of sets:

$$A^E = \{a_1^E, a_2^E\}, \quad (26)$$

where  $a_1^E$  – climatic conditions;  $a_2^E$  – economic and market conditions.



The next step is to build a mathematical model for monitoring the quality, risks, and effectiveness of the project.

1. The set of input parameters ( $X$ ):

$$X = \{G, P, R, O, S, Z, K, L, A\}, \quad (27)$$

where  $G$  – material resources;  $P$  – human resources;  $R$  – software resources;  $I$  – information resources;  $O$  – stakeholders;  $S$  – services supporting IT infrastructure (SI);  $W$  – technologies for developing and testing the project product;  $K$  – project parameters;  $L$  – project product parameters;  $A$  – external environment parameters of the project.

2. The set of output parameters ( $Y$ ):

$$Y = \{Q, R, E, H\}, \quad (28)$$

where  $Q$  – project quality;  $R$  – project risks;  $E$  – project efficiency;  $H$  – communication channels between the elements of the IT project management model.

3. The number of direct links is represented as a set:

$$H^P = \{h_1^P, h_2^P, h_3^P, h_4^P\}, \quad (29)$$

Direct links between all the elements of the model. Assume that direct links include communication between the main resources, for example, 10 direct links.

4. The number of feedback links is represented as a set:

$$H^{Zv} = \{h_1^{Zv}, h_2^{Zv}, h_3^{Zv}, h_4^{Zv}\}, \quad (30)$$

Feedback links between all elements of the model. Assume that there are 5 feedback links. The set of input parameters for the project model ( $X$ ) is represented as:

$$X = \{x_i, i=1, 2, \dots, N_i\}. \quad (31)$$

The number of knowledge areas in the model is  $M_i$ . Assume that  $N_i=22$ .

The project quality assessment ( $Q$ ) is defined as:

$$Q = \sum_{i=1}^{22} \sum_{j=1}^{TP} \sum_{k=1}^{10} (Q_1(x_i, t_j) + Q_2(h_k)) \quad (32)$$

under constraint:

$$\forall (x_i \in X) (a_i \in Q) \exists h_i \in TP, TP \geq 0 \text{ ma } Q \geq Q_{min}, Q_{min} \geq 0, \quad (33)$$

where  $Q_{min}$  – the minimum quality level of the project;  $Q_1$  – the quality assessment function based on the creation of input parameters from  $X$  at time  $t_j \in TP$ ;  $Q_2$  – the quality assessment function for communication channels between elements of the model from  $X$ .

The project risk assessment ( $R$ ) is defined as:

$$R = \sum_{i=1}^{22} \sum_{j=1}^{TP} \sum_{k=1}^{10} (R_1(x_i, t_j) + R_2(h_k)) \quad (34)$$

under constraint:

$$\forall (x_i \in X) (a_i \in Q) \exists h_i \in TP, TP \geq 0 \text{ ma } R \leq R_{max}, R_{max} \geq 0, \quad (35)$$

where  $R_{max}$  – the maximum allowable level of project risks;  $R_1$  – the risk assessment function based on the creation of input parameters from  $X$  at time  $t_j \in TP$ ;  $R_2$  – the risk assessment function for communication channels between elements of the model from  $X$ .

The project effectiveness assessment ( $E$ ) is defined as:

$$E = \sum_{i=1}^{22} \sum_{j=1}^{TP} \sum_{k=1}^{10} (E_1(x_i, t_j) + E_2(h_k)) \quad (36)$$

under constraint:

$$\forall (x_i \in X) (a_i \in \Omega) \exists h_i \in TP, TP \geq 0 \text{ ma } E \geq E_{\min}, E_{\min} \geq 0 \quad (37)$$

where  $E_{\min}$  – the minimum level of project effectiveness;  $E_j$  – the effectiveness assessment function based on the creation of input elements from  $X$  at time  $t_j \in TP$ ;  $E_j$  – the effectiveness assessment function for communication channels between elements of the model from  $X$ .

Example values: 1. *Material resources*:  $x_1$  – FC28 sensor;  $x_2$  – ESP8266 microcontroller;  $x_3$  – irrigation system. 2. *Human resources*:  $x_4$  – project manager;  $x_5$  – system architect;  $x_6$  – designer;  $x_7$  – tester;  $x_8$  – developer;  $x_9$  – analyst;  $x_{10}$  – agricultural consultant;  $x_{11}$  – documentation manager;  $x_{12}$  – accountant. 3. *Software resources*:  $x_{13}$  – Python programming language;  $x_{14}$  – SQL database;  $x_{15}$  – web server. 4. *Information resources*:  $x_{16}$  – data from humidity sensors;  $x_{17}$  – data on types of grain crops;  $x_{18}$  – GIS (Geographic Information System);  $x_{19}$  – technical documentation and manuals. 5. *Services*:  $x_{20}$  – monitoring and management of the humidity sensor system;  $x_{21}$  – storage and analysis of sensor data;  $x_{22}$  – web server support.

For each resource and project component, functions evaluating quality, risk, and efficiency are calculated based on their specific values and duration of use. Furthermore, corresponding functions are defined for the communication channels linking the various project elements.

This approach enables a comprehensive analysis of the project's quality, risks, and efficiency, providing a robust foundation for effectively managing and optimizing the development of the IoT system designed for soil moisture monitoring.

**Conclusions.** The integration of set theory as a mathematical framework for developing an IoT system for automated soil moisture monitoring offers a groundbreaking approach in agricultural technology. By utilizing the foundational principles of set theory, the complex interactions among critical system elements – such as material and human resources, software and informational assets, stakeholders, IT infrastructure support services, development and testing technologies, project parameters, product attributes, and external environmental factors – have been systematically modeled. This structured methodology enhances the precision of system design and significantly improves decision-making processes in project management, paving the way for more efficient and scalable solutions.

The proposed IoT system demonstrates significant potential for optimizing irrigation, rationalizing resource usage, reducing costs, and minimizing environmental impact. Moreover, it promotes the integration of modern technologies into agriculture, which is critically important in addressing global challenges such as climate change and water scarcity.

Future research prospects include expanding mathematical models to account for additional parameters, such as forecasting soil moisture dynamics using machine learning, and developing multifunctional IoT systems capable of monitoring temperature, light levels, and soil acidity. Special attention should also be directed toward integrating IoT systems with big data (Big Data) and artificial intelligence to enhance the adaptability and autonomy of systems in real time.

In conclusion, this research not only establishes a theoretical foundation for further improvement of IoT solutions but also paves the way for groundbreaking automation advancements that will significantly impact the development of agricultural technologies.

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