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A DECOMPOSITION-BASED APPROACH TO MULTI-OBJECTIVE UNMANNED VEHICLE ROUTE OPTIMISATION

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The rapid integration of unmanned vehicles (UVs) into industries such as logistics, defence, agriculture, and environmental monitoring has necessitated the development of advanced route optimisation methodologies to enhance operational efficiency. Given their ability to operate in hazardous environments and under diverse meteorological conditions, UVs offer significant advantages over traditional transportation methods. However, optimising UV routes presents a complex challenge due to the need to balance multiple, often conflicting, objectives, including minimising travel distance and time, reducing fuel consumption, ensuring safety in varying weather conditions, adapting to terrain constraints, and prioritising mission-critical tasks. This study addresses the problem of multi-objective UV route optimisation by introducing a decomposition-based approach that divides the optimisation process into two stages: (1) the formation of a subset of candidate routes based on predefined constraints and (2) the selection of the optimal route from this subset using a combination of the PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) method and heuristic algorithms. The integration of these techniques enables effective decision-making by systematically ranking alternative routes according to multiple evaluation criteria. The proposed methodology efficiently reduces computational complexity, making it particularly suitable for large-scale UV deployment scenarios. A comparative analysis of different optimisation strategies demonstrates the effectiveness of the proposed approach. The results indicate that the method reduces computational time and resource consumption while maintaining flexibility in dynamic environments. By leveraging the PROMETHEE method for multi-criteria decision-making and heuristic search techniques for rapid optimisation, the study provides a practical solution for UV route planning, ensuring enhanced adaptability to operational constraints. The findings contribute to ongoing research in UV logistics and mission planning by offering a structured framework that balances efficiency, reliability, and computational feasibility in complex, multi-objective optimisation tasks.

Key words: *unmanned vehicles (UVs), multi-objective optimization, PROMETHEE, Branch-and-Bound Method, decomposition of problem.*

Симонов Д. І., Симонов Є. Д., Заіка Б. Ю. Декомпозиційний підхід до багатокритеріальної оптимізації маршрутів безпілотних транспортних засобів

Швидка інтеграція безпілотних транспортних засобів (БТЗ) у такі галузі, як логістика, оборона, сільське господарство та екологічний моніторинг, зумовила необхідність розробки передових методів оптимізації маршрутів для підвищення операційної ефективності. Завдяки здатності працювати в небезпечних умовах та за різноманітних метеорологічних обставин, БТЗ мають значні переваги над традиційними методами транспортування. Проте оптимізація маршрутів БТЗ є складним завданням, оскільки вимагає збалансування численних, часто суперечливих цілей, включаючи мінімізацію відстані й часу подорожі, зниження витрат пального, забезпечення безпеки за різних погодних *умов, адаптацію до обмежень рельєфу та пріоритезацію критично важливих завдань. У цьому дослідженні розглянуто проблему багатокритеріальної оптимізації маршрутів БТЗ шляхом впровадження підходу, заснованого на декомпозиції, що розділяє процес оптимізації на два етапи: (1) формування підмножини кандидатних маршрутів на основі заданих обмежень і (2) вибір оптимального маршруту з цієї підмножини з використанням комбінації методу PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) та евристичних алгоритмів. Інтеграція цих методів забезпечує ефективне прийняття рішень шляхом систематичного ранжування альтернативних маршрутів за кількома критеріями оцінювання. Запропонована методологія дозволяє зменшити обчислювальну складність, що робить її особливо придатною для сценаріїв масштабного розгортання БТЗ. Порівняльний аналіз різних стратегій оптимізації демонструє ефективність запропонованого підходу. Отримані результати свідчать, що метод скорочує час обчислень і витрати ресурсів, забезпечуючи при цьому гнучкість у динамічних середовищах. Використовуючи метод PROMETHEE для багатокритеріального прийняття рішень та евристичні методи пошуку для швидкої оптимізації, дослідження пропонує практичне рішення для планування маршрутів БТЗ, забезпечуючи покращену адаптацію до операційних обмежень. Отримані результати роблять внесок у подальші дослідження в галузі логістики та планування місій БТЗ, пропонуючи структурований підхід, що балансує між ефективністю, надійністю та обчислювальною доцільністю в складних задачах багатокритеріальної оптимізації.*

Ключові слова: *безпілотні транспортні засоби (БТЗ), багатокритеріальна оптимізація, PROMETHEE, метод гілок і меж, декомпозиція задачі.*

Introduction. In recent years, the utilisation of unmanned vehicles (UVs) has undergone substantial expansion across a range of sectors, including defence, agriculture, logistics, and environmental monitoring. This accelerated growth can be attributed to their distinct advantages, including their exceptional agility, aptitude for operation in perilous environments, and their capacity to function effectively under a wide range of meteorological conditions. As UV adoption increases, there is a pressing need for advanced methodologies to effectively plan and optimize their routes. The implementation of such optimisation techniques has the potential to enhance operational efficiency and ensure seamless coordination of vehicles.

This study focuses on the multi-objective optimization of motor vehicle routes. It explores optimization methods that account for conflicting criteria and constraints specific to UVs, such as distance, time, weather conditions, terrain, fuel consumption, risk factors, and task prioritization. Such constraints include, but are not limited to, distance, time, weather conditions, terrain, fuel consumption, risk factors and task prioritization. The study pays particular attention to the application of the PROMETHEE method (Preference Ranking Organization Methods for Enrichment Evaluation) and heuristic algorithms for determining optimal routes and assigning them to specific vehicles.

The aim of the study. The primary objective of this research is to formulate and validate effective methodologies for multi-objective vehicle route optimisation, with the goal being to enhance efficiency and safety across a range of applications.

Analysis of recent research and publications. The optimisation of UV routes is a complex challenge that involves balancing multiple conflicting criteria and constraints [1]. Key factors in this process include distance, time, weather conditions, terrain, fuel consumption, collision risks, and task priority. Addressing this challenge necessitates the application of sophisticated mathematical models and algorithms to identify optimal or near-optimal solutions within the framework of Pareto-optimal search problems. Given the classification of multi-criteria UV route optimization as an NP-hard problem, conventional algorithms often prove ineffective due to the complexity and large-scale nature of the data involved [2].

Shevchuk et al. developed a transport route optimisation method using a neural network trained on real or simulated data to estimate travel time and identify the fastest route. Numerical experiments validated the method across 32 scenarios, determining the optimal network structure and the minimal required training dataset size [1]. Bahaeddin Türkoğlu and Hasan Eroğlu explored the use of genetic algorithms in route optimisation and demonstrated their effectiveness in solving complex problems such as the travelling salesman and driver route planning. A key focus in their study was on the crossover operator, which improved solutions by exploiting weaker candidates, with a case study on optimising the routes of power transmission lines [2]. Xiuli Li investigated the optimisation of distribution routes for convenience store chains using an enhanced genetic algorithm and demonstrated its superiority over the classical approach. The study shows how this improvement can be practically applied to improve distribution efficiency and economic benefits for convenience store chains [3].

In conclusion, developing innovative optimisation methods is essential for minimising resource consumption while maintaining solution quality, especially given the increasing complexity of modern transportation and logistics. The effectiveness of heuristic methods, genetic algorithms, and machine learning has been demonstrated in solving complex routing problems, enabling efficient identification of near-optimal solutions. As route optimisation remains a highly relevant challenge, these approaches not only enhance distribution and transport planning but also offer valuable applications in fields such as UV deployment, logistics, and emergency response.

Multi-objective route optimisation problem. The multi-objective UV route optimisation problem is a complex task that requires consideration of many different criterions $X = (X_1 \cup X_2)$ to determine the best route allocation [4]. The main goal is to find a balanced solution that satisfies the various requirements and constraints imposed on UVs.

Optimisation objectives can be divided into two subsets: those that maximise $f\left(x_i^1\right)$, and those that minimise $f\left(x_i^2\right)$, where $X_1 = \left\{x_i^1\right\}, X_1 \subseteq X$. The first subset $f\left(x_i^1\right)$ includes criterions that demonstrate the desire to maximise the usefulness of the system or the efficiency of the route. Among them are:

– reliability of communication: ensuring stable communication between the vehicle and the dispatcher (server);

– bandwidth: maximising the number of deliveries or transported goods per unit of time;

– quality of service: increasing customer satisfaction through timely deliveries.

The second subset $f\left(x_i^2\right)$ includes criterions whose indicators are minimised, where $X_2 = \left\{x_i^2\right\}, X_2 \subseteq X$. The indicators of the second group include:

– route time: minimising the time required to complete a task;

– resource consumption: reduction of fuel or electricity costs;

– risk of accidents: decrease a likelihood of emergencies;

– environmental impact: reduction of harmful substances and noise emissions.

The formulation of a multi-objective optimisation problem involves the formulation of a mathematical model that takes into account these objectives and their interactions with each other [5]. This involves determining the objective function constraints for each criterion and methods for weighting the criteria and for finding trade-off solutions.

The mathematical model of the multi-objective APC route optimisation problem can be formulated as follows:

$$\begin{cases} \omega_i^1 \cdot f\left(x_i^1\right) \to \min \\ \omega_i^2 \cdot f\left(x_i^2\right) \to \max \end{cases}, \quad (1)$$

by the constraints

$$\sum_j g\left(x_{ii}^1\right) \le b_i^1, \tag{2}$$

$$\sum_j g\left(x_{ii}^2\right) \ge b_i^2, \tag{3}$$

$$\sum_i \omega_i^1 = 1, \sum_i \omega_i^2 = 1, \tag{4}$$

$$x_{ii}^1, x_{ii}^2 \ge 0, i = \overline{1,n}, j = \overline{1,m}, \tag{5}$$

where ω_i^1, ω_i^2 are the weighting criteria coefficients of the set X_1 and X_2 respectively; $g\left(x_{ii}^1\right), g\left(x_{ii}^2\right)$ are the functions for determining the actual indicators of the functioning criteria; b_i^1, b_i^2 are the existing system constraints.

Due to the NP-complexity of problems (1)-(5), the search for an optimal solution requires significant computational resources, which complicates the use of traditional optimisation methods. The stage-by-stage decomposition of the solution search allows us to divide a complex problem into smaller subtasks, which significantly simplifies the computation process and increases the efficiency of finding an optimal solution.

Decomposition of a multi-objective route optimisation problem. Decomposition of the route optimisation problem is a promising approach to reduce the complexity of NP-complete problems. It allows dividing the main problem into several subproblems, each of which is solved sequentially, taking into account the relevant criteria and constraints and the results of the previous stage. This provides a more structured approach and reduces the overall computational complexity. The decomposition consists of the two following stages.

1. Formation of a subset of candidate routes: a preliminary selection of routes that meet basic requirements, such as maximum length, route duration, budget constraints, etc. The result is a set of potential routes that satisfy the initial requirements and eliminate unacceptable options.

2. Optimal route selection: from the resulting subset, the route that best meets the priorities and additional criteria, such as minimising total costs, maximising safety, or taking into account climatic conditions, is selected.

This approach divides a complex problem into two simpler ones, making it easier to find the optimal solution. Decomposition not only improves computational efficiency, but also makes it easier to adapt to changing conditions and new constraints, which is important for dynamic optimisation systems [6]. This approach increases the flexibility and adaptability of the route planning process, contributing to more efficient resource management and optimal results.

Size reduction of the alternatives set. The analysis of alternatives to determine the optimal UV route is an important task, especially in the context of the development of autonomous transport technologies and their application in various fields, including transportation, military use, geological survey, agriculture, and environmental monitoring. The multi-criteria nature of alternatives and the NP-complexity of these tasks make it necessary to use methods that allow to systematise these criterions and evaluate alternatives with regard to all aspects. The use of such methods reduces the complexity of decision-making, as they provide a structured approach to evaluating and comparing alternatives. This is especially important in the face of numerous options, when it is difficult to choose the best solution without the use of specialised methods.

Methods such as PROMETHEE provide tools for effective analysis of multi-criteria problems [7]. They allow to structure the criteria by importance and determine their impact on the final decision. This helps to avoid subjective influence in the evaluation of alternatives and ensure objective results. Given the increasing complexity of UV route selection and the requirements for accuracy and efficiency, the use of alternative analysis methods, such as PROMETHEE, is essential to ensure optimal selection and successful mission execution.

The application of the PROMETHEE method allows selecting a subset of alternative solutions based on the following parameters [8].

1. Multi-criteria decision: PROMETHEE allows to evaluate alternatives according to different criterions and take into account their importance.

2. Criteria conflicts: PROMETHEE allows to balance conflicts of criterions by assigning a weight to each of them.

3. Uncertainty and risk: PROMETHEE allows to take into account the uncertainty of conditions and risks when making a decision [9].

4. Comprehensiveness of the analysis: PROMETHEE allows for a comprehensive analysis of alternatives, taking into account different aspects.

5. Decision support: PROMETHEE provides tools for objective decision-making based on data, which reduces the influence of subjective preferences.

These parameters make methods such as PROMETHEE useful and necessary for analysing alternatives when selecting the optimal route for UVs. Accordingly, the PROMETHEE method is a multi-criteria decision-making method that allows ranking alternatives based on their relative superiority according to several criteria [10]. The main idea is to compare each alternative with all others based on a set of criterions and make a decision based on these comparisons.

To solve the first stage of choosing the optimal distribution of UV routes, the PROMETHEE algorithm can be as follows.

1. Determination of the set of alternatives $A=\{a_i \mid i=1,...,n\}$ and the set of criterions $X=\{x_j \mid j=1,...,m\}$ by which the alternatives will be evaluated. The set consists of two subsets X_1 and X_2, i.e. $X=\left(X_1 \cup X_2\right)$. Therefore, for a generalised analysis by the PROMETHEE method, it is necessary to transform the functions to a single format. For example, the criterions of both subsets X_1 and X_2 must be minimised by performing the transformation $\max\left\{f(x)\right\} \Rightarrow \min\left\{-f(x)\right\}$.

2. Determination of criteria weights ω_j under the condition $\sum \omega_j = 1$. The weights of the criteria are determined by experts based on their importance for decision making.

3. Normalisation of the data matrix. Each alternative score for each criterion is normalised against the maximum possible score and the minimum possible score by the following formula:

$$v_{ij}^{norm} = \frac{v_j(a_i) - v_j^{\min}(a_i)}{v_j^{\max}(a_i) - v_j^{\min}(a_i)}. \quad (6)$$

4. Evaluation of the superiority function of a_i over a_k based on criterion a_j:

$$H_j(a_i,a_k) = H_j(f_j(a_i,a_k)), \quad (7)$$

$$H_j(a_i,a_k) \in [0,1], i=\overline{1,n}, k=\overline{1,n}, k \neq i, j=1,...,m, \quad (8)$$

where $f_j(a_i,a_k) = v_j(a_i) - v_j(a_k)$.

Suppose that the superiority function has a linear shape (V-shape), then $H_j(a_i,a_k)$ can be calculated with the formula:

$$H_j(a_i,a_k)=\begin{cases}1,\text{if } v_j()-v_j(a_k)>p_j\\ \dfrac{v_j(a_i)-v_j(a_k)}{p_j},\text{if } 0<v_j(a_i)-v_j(a_k)\le p_j\\ 0,\text{if } v_j(a_i)-v_j(a_k)\le 0\end{cases},\ i=\overline{1,n},k=\overline{1,n},k\ne i, \quad (9)$$

where p_j is the satisfactory level of the superiority threshold.

5. Evaluation of the weighted preference function of a_i over a_k based on criterion a_j:

$$F(a_i,a_k)=\sum_{j=1}^{m} w_j^c\cdot H_j(a_i,a_k),\forall(a_i,a_k), \quad (10)$$

$$F(a_i,a_k)\in[0,1],\ F(a_k,a_k)=0,\ i=\overline{1,n},k=\overline{1,n},k\ne i. \quad (11)$$

6. Flow assessment for each alternative a_i:

6.1. The output flow $\Phi^+(a_i)$ is the sum of the positive differences between the scores of alternative a_i and every other alternative a_k across all criterions x_j, where $i=\overline{1,n},k=\overline{1,n},k\ne i$. Thus, the output flow reflects the extent to which an alternative a_i dominates the other alternatives a_k.

$$\Phi^+(a_i)=\tfrac{1}{n-1}\sum_{i\ne k}F(a_i,a_k),\ i=\overline{1,n},k=\overline{1,n}. \quad (12)$$

6.2. The input flow $\Phi^-(a_i)$ is the sum of the negative differences between the scores of the other alternatives a_k and the scores of the alternative a_i across all criterions x_j, where $i=\overline{1,n},k=\overline{1,n},k\ne i$. Accordingly, the input stream reflects the extent to which the other alternatives a_k dominate the alternative a_i.

$$\Phi^-(a_i)=\tfrac{1}{n-1}\sum_{i\ne k}F(a_k,a_i),\ i=\overline{1,n},k=\overline{1,n}. \quad (13)$$

6.3. Net flow is the difference between the output $\Phi^+(a_i)$ and input $\Phi^-(a_i)$ flow for each alternative a_i. The formula of net flow is as follows:

$$\Delta\Phi(a_i)=\Phi^+(a_i)-\Phi^-(a_i). \quad (14)$$

7. Determining the superiority level of alternatives:

$$a_i\mathrm{I}a_k \text{ if } (\Phi^+(a_i)=\Phi^+(a_k))\wedge(\Phi^-(a_i)=\Phi^-(a_k)), \quad (15)$$

$$a_i\mathrm{P}a_k \text{ if }\begin{cases}(\Phi^+(a_k)\ge\Phi^+(a_l))\wedge(\Phi^-(a_k)<\Phi^-(a_l))\\ (\Phi^+(a_k)>\Phi^+(a_l))\wedge(\Phi^-(a_k)=\Phi^-(a_l))\end{cases}, \quad (16)$$

$$a_k\mathrm{Q}a_l - \text{others}. \quad (17)$$

Example 1. Using the input data (Table 1), determine the priority subset of alternatives using the PROMETHEE method, provided that $\left|A^{prom}\right|=10$.

Table 1

Input data

a_i	Normalised criterion score for alternatives x_j									
	1	2	3	4	5	6	7	8	9	10
1	0,9	0,9	0,5	1	0,5	0,7	0,8	0,6	1	0,7
2	1,0	0,7	1	0,7	1	0,3	0,5	0,5	0,6	0,8
3	0,5	0,8	0,9	0,9	0,5	0,9	0,7	0,7	1	0,8
4	0,8	0,9	0,8	0,6	0,9	0,6	0,8	0,6	0,6	0,7
5	0,4	0,6	0,4	0,3	0,6	0,2	0,5	0,4	0,9	0,8
6	0,4	0,5	0,9	0,4	0,8	0,3	0,9	0,8	0,5	0,8
7	0,9	0,6	0,9	0,5	0,5	0,7	0,9	0,7	0,5	0,7
8	0,4	0,9	0,8	0,4	0,3	0,6	0,3	0,3	0,7	0,4
9	0,6	0,7	0,7	0,9	0,9	0,9	0,6	0,9	0,6	0,8
10	0,8	0,5	1	0,7	0,6	0,6	0,9	0,6	0,8	0,3
11	0,9	0,4	0,6	1	0,4	0,5	0,9	0,6	0,6	0,5
12	0,5	1,0	0,8	0,7	0,8	0,6	0,6	0,3	0,5	0,9
13	0,3	0,5	0,8	0,6	0,5	0,3	0,6	0,9	0,7	0,3
14	1	0,9	0,8	0,4	0,4	0,4	0,2	0,7	0,2	0,2
15	0,7	0,7	0,3	1	0,6	0,8	0,7	1	0,5	0,5
Criterion cut-off threshold p_j	0,2	0,2	0,1	0,1	0,2	0,1	0,1	0,2	0,2	0,1
Weight of the criterion ω_j	0,3	0,12	0,07	0,14	0,04	0,07	0,11	0,04	0,09	0,02

Based on (12)-(13), the weighted superiorities of the alternatives and the input and output flows are calculated. The results are shown in Table 2.

Table 2

The weighted superiorities of alternatives and flows

a_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\Phi^+(a_i)$
1	–	0,57	0,41	0,51	0,9	0,76	0,39	0,78	0,72	0,65	0,32	0,71	0,79	0,42	0,65	0,61
2	0,32	–	0,4	0,47	0,7	0,68	0,59	0,71	0,39	0,48	0,45	0,51	0,58	0,47	0,49	0,52
3	0,26	0,52	–	0,52	0,81	0,67	0,49	0,77	0,45	0,53	0,39	0,51	0,82	0,44	0,43	0,54
4	0,11	0,34	0,41	–	0,89	0,69	0,29	0,65	0,58	0,18	0,29	0,53	0,61	0,4	0,57	0,47
5	0,04	0,11	0,04	0,11	–	0,18	0,13	0,29	0,11	0,15	0,27	0,13	0,29	0,26	0,17	0,16
6	0,18	0,15	0,08	0,18	0,37	–	0,09	0,27	0,2	0,1	0,2	0,24	0,39	0,36	0,23	0,22
7	0,12	0,22	0,3	0,39	0,73	0,51	–	0,67	0,48	0,37	0,3	0,55	0,63	0,37	0,49	0,44
8	0,07	0,25	0,07	0,06	0,33	0,28	0,21	–	0,24	0,13	0,29	0,09	0,4	0,16	0,28	0,20
9	0,19	0,3	0,28	0,26	0,78	0,73	0,44	0,72	–	0,43	0,31	0,53	0,65	0,44	0,23	0,45
10	0,14	0,31	0,38	0,28	0,73	0,66	0,31	0,76	0,57	–	0,31	0,61	0,62	0,46	0,47	0,47
11	0,16	0,36	0,34	0,49	0,73	0,67	0,3	0,62	0,52	0,45	–	0,65	0,68	0,4	0,54	0,49
12	0,2	0,26	0,18	0,16	0,69	0,55	0,32	0,58	0,2	0,18	0,29	–	0,58	0,47	0,25	0,35
13	0,11	0,18	0,08	0,15	0,45	0,27	0,24	0,42	0,17	0,17	0,27	0,17	–	0,43	0,2	0,24
14	0,34	0,23	0,37	0,41	0,67	0,49	0,39	0,43	0,52	0,53	0,46	0,41	0,49	–	0,56	0,45
15	0,06	0,35	0,36	0,25	0,73	0,67	0,31	0,71	0,36	0,39	0,23	0,6	0,74	0,44	–	0,44
$\Phi^-(a_i)$	0,16	0,30	0,26	0,30	0,68	0,56	0,32	0,60	0,39	0,34	0,31	0,45	0,59	0,39	0,40	

Based on (14), the net flow $\Delta\Phi(a_i)$ is calculated (Table 3).

Table 3

The net flow

a_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\Delta\Phi(a_i)$	0,45	0,22	0,28	0,16	-0,5	-0,3	0,12	-0,4	0,06	0,13	0,18	-0,1	-0,4	0,06	0,05

Table 3 and the terms of the problem suggest that a subset of alternatives $A^{prom} \subseteq A$, under the condition $\left|A^{prom}\right| = 10$, will have the form $A^{prom} = \{a_1, a_2, a_3, a_4, a_7, a_9, a_{10}, a_{11}, a_{14}, a_{15}\}$.

Setting the route assignment. The second step in solving the UV route optimisation problem is to assign alternative route options to certain UVs. One of the options for solving the assignment problem is the branch-and-bound method [11]. The branch-and-bound method allows you to efficiently find optimal solutions in discrete optimisation problems, avoiding a complete search of all possible options. This method guarantees finding an exact solution, while significantly reducing the computation time compared to a full search. However, obtaining an exact solution to the UV route optimisation problem using the branch-and-bound method requires large computing resources, as well as a significant amount of memory to store the branching tree. In addition, each step in the branching process of the algorithm is accompanied by an estimate of the deviation of the approximate solution from the optimal solution. If an algorithm is used with a focus on finding ε-approximate solutions from the very beginning, this can lead to increased filtering, which in turn will reduce the amount of information and, accordingly, reduce the number of problems to be solved [12].

Since at the first stage of solving the optimisation problem, a subset $A^{prom} \subseteq A$ was determined, the choice of priority routes a_i that form a subset A_i', will be performed from the set A^{prom}, respectively, $A_i' \subseteq A^{prom}$. The choice of the route implies the priority selection of alternatives with the highest net flow $\Delta\Phi(a_i)$. Accordingly, the objective function for selecting a priority route will be:

$$f(x) = \sum_{i=1}^{n} \Delta\Phi(a_i) \cdot r_i \to \max, \tag{18}$$

under the conditions

$$\sum_{i=1}^{n} l_i \cdot r_i + l^{finish} \leq l_{TTC}, \tag{19}$$

$$r_i = \begin{cases} 1, \text{if } a_i \in A_i' \\ 0, \text{if } a_i \notin A_i' \end{cases}, \tag{20}$$

$$\Delta\Phi(a_i) > 0, l_i, l_{TTC} > 0,\ a_i \geq a_{i+1} \geq \ldots \geq a_n, \tag{21}$$

where l_1 is the distance from the location of the route start to the location of the alternative $a_i \in A^{prom}$; l_i is the distance from the location of the alternative a_i to the location of the alternative $a_{i+1}, i = \overline{2, n}$; l^{finish} is the distance from the last covered location of the alternative $a_i \in A^{prom}$; to the location of the route start; l_{TTC} is the maximum UV movement distance determined by tactical and technical characteristics.

An algorithm for finding a set of optimal routes, given the UV's constraints (mandatory return to the launch location with an error of ε) and the decision maker's priorities:

1. $l_{TTC} :\neq 0, l_i :\neq 0, l^{finish} :\neq 0, \Delta\Phi(a_i) \neq 0, n := 0, m := 0.$
2. Find the vertex: $a_i = \max\{\Delta\Phi(a_i)\}, a_i \in A^{prom}, \left(l_i + l^{finish}\right) \leq l_{TTC}, i = 1 + n.$
3. $A_i' = A_i' \cup \{a_1\}.$
4. $A^{prom} = A^{prom} \setminus A_i'.$

5. If $\left(l_{TTC} - l_1 - l^{finish}\right) \geq \min\left\{l_i \mid a_i \in A^{prom},\ i = \overline{2,n}\right\}$, then $n := +1$, and go to step 2.

6. If $A^{prom} \neq \varnothing$, then $m := +1$, and go to step 2.

7. End of the algorithm.

Example 2. Assume that $l_{TTC} = 25\,км.$ Based on the results of Example 1, determine the optimal routes and number of UVs required to cover a subset of A^{prom}. Distance parameters are shown in Tables 4 and 5. Graph model of alternative UV routes is presented in Figure 1.

Table 4

Parameters of distance from the UV launch point and net flow

	a_i									
	1	**2**	**3**	**4**	**7**	**9**	**10**	**11**	**14**	**15**
$\Delta\Phi(a_i)$	0,45	0,22	0,28	0,16	0,12	0,06	0,13	0,18	0,06	0,05
l_i	9,1	6,8	5,4	8,1	7,6	5,1	3,9	9,8	7,8	6,1

Table 5

Parameters of distance between vertexes

a_i	**1**	**2**	**3**	**4**	**7**	**9**	**10**	**11**	**14**	**15**
1	0	5,4	7,2	2,4	3,1	1,8	0,9	7,6	9,1	2,1
2	5,4	0	1,5	2,5	6,1	4,9	8,3	7,3	6,5	3,7
3	7,2	1,5	0	3,1	3,9	2,7	2,8	0,9	7,3	8,1
4	2,4	2,5	3,1	0	1,8	3,9	7,6	3,3	6,2	7,1
7	3,1	6,1	3,9	1,8	0	8,1	6,7	3,4	4,9	8,1
9	1,8	4,9	2,7	3,9	8,1	0	3,1	5,2	7,1	3,4
10	9,9	8,3	2,8	7,6	6,7	3,1	0	3,2	6,1	8,1
11	7,6	7,3	8,9	3,3	3,4	5,2	3,2	0	3,7	2,9
14	9,1	6,5	7,3	6,2	4,9	7,1	6,1	3,7	0	5,1
15	2,1	3,7	8,1	7,1	8,1	3,4	8,1	2,9	5,1	0

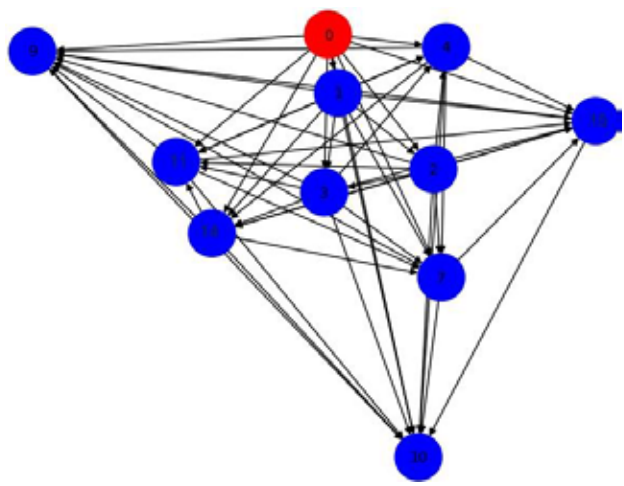

Fig. 1. Graph model of alternative UV routes

To calculate the optimal routes and the number of UVs, equations (18)-(22) and the above algorithm for finding the set of optimal routes are used. The results are shown in Figure 2 and Table 6.

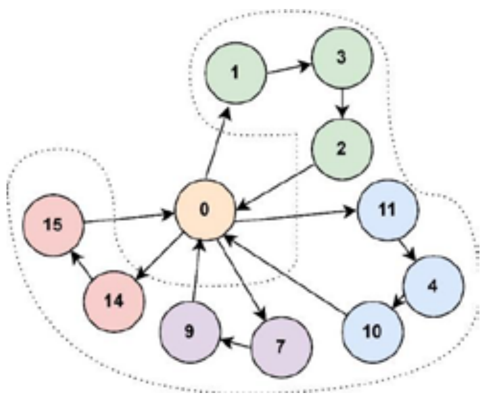

Fig. 2. Model of UV optimal routes

Table 6

Results of route distribution

Підмножина $A_i' \subseteq A^{prom}$	**Маршрут**	$\sum_i \Delta\Phi(a_i)$	$\sum_i l_i$ **, км**
$A_1' = \{a_1, a_2, a_3\}$	$a_0 \to a_1 \to a_3 \to a_2 \to a_0$	0,95	24,6
$A_2' = \{a_4, a_{10}, a_{11}\}$	$a_0 \to a_{11} \to a_4 \to a_{10} \to a_0$	0,47	24,6
$A_3' = \{a_7, a_9\}$	$a_0 \to a_7 \to a_9 \to a_0$	0,18	20,8
$A_4' = \{a_{14}, a_{15}\}$	$a_0 \to a_{14} \to a_{15} \to a_0$	0,11	19,0

Based on results, four units of UVs are required to complete the task of covering the vertices of the graph (each taking one of four alternative routes).

Evaluating the method effectiveness. To evaluate the effectiveness of the proposed method, it is advisable to compare its operation with the most common method of route allocation – the method of the quadratic assignment problem. Such a comparison will allow us to identify the advantages and disadvantages of each approach in real-world applications.

The Quadratic Assignment Problem (QAP) method is widely used to solve complex combinatorial optimisation problems where it is necessary to distribute n objects to n locations in such a way as to minimise the total cost. The popularity of this method is due to its ability to accurately model various real-world situations, such as optimising the location of production facilities, placing electronic components, planning medical facilities, etc. The advantages of the QAP method include the following.

1. Versatility: QAP can be applied to a wide range of problems where an optimal assignment needs to be found.

2. Modelling accuracy: QAP allows to take into account the complex relationships between objects and places, which makes it useful for real systems.

3. Flexibility: the ability to adapt the method to different types of problems and take into account various constraints.

4. Sophistication: there are many algorithms and software for solving QAP.

The disadvantages of the QAP method include the following.

1. High computational complexity: QAP belongs to the class of NP-hard problems, which makes its solution extremely time-consuming for large problem sizes.

2. Limited scalability: for large problems, the number of possible combinations grows exponentially, making traditional methods inefficient.

3. The need for approximate methods: due to computational limitations, it is often necessary to use heuristic or metaheuristic methods that do not always guarantee an optimal solution.

The algorithms were compared by their execution time on different sizes of the input matrix. Testing was conducted on modern hardware using an Intel Core i7 processor, 32 GB of RAM, and a solid-state drive. Matrices of different sizes from 3x3 to 500x500 were used, which allowed to evaluate the performance of the algorithms on different problem scales. This experiment revealed how the proposed method copes with small and large problems compared to the quadratic assignment problem method. The results are shown in Figure 3.

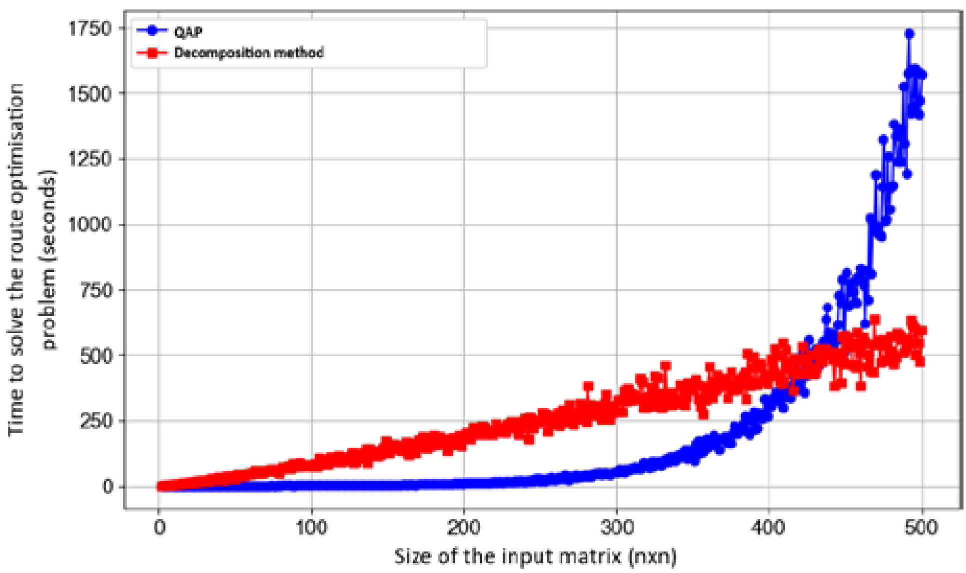

Fig. 3. Results of comparing the task solution time

As can be seen in Figure 3, the proposed method of decomposition of the multicriteria optimization problem of unmanned vehicle routes is a more efficient method compared to the method of the quadratic assignment problem on large-size matrices. The nature of the growth in the need for resources of the decomposition method is almost linear, which allows predicting the necessary resources for solving optimisation problems.

Conclusions. The paper considers the problem of multi-objective optimisation of UV routes. A method of decomposition of the multiple-objective UV route optimisation problem is proposed, which allows dividing it into two stages: formation of a subset of candidate routes and selection of the optimal route from this subset.

The considered examples of choosing the optimal route distribution and route assignment and comparative analysis demonstrate the effectiveness of the proposed method. The obtained results demonstrate that the proposed approach allows reducing resource costs and find optimal solutions faster, ensuring a balance between the quality of results and the cost of time and resources, especially for large input data sizes. This is achieved by reducing the computational complexity of the optimisation problem and the ability to quickly adapt to changing conditions.

Prospects for further research include improving existing methods and developing new approaches to multi-objective optimisation, including the use of distributed algorithms. This will further increase the efficiency of UVs in various fields of activity, such as logistics, emergency services and environmental monitoring, ensuring more efficient resource management and achieving optimal results.

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