СИСТЕМНИЙ АНАЛІЗ

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A STUDY OF THE MARKET OF THREE INTERCHANGEABLE GOODS FOR SUSTAINABILITY

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This paper presents a study of the market dynamics of three interchangeable goods based on a mathematical model that takes into account the interaction between sellers, buyers and the state. The stability of the equilibrium state of the system for different values of the model parameters is analyzed. Analytical expressions for conditions of equilibrium stability were obtained. The impact of demand elasticity and taxation on market stability is studied. It is shown that, under certain conditions, even a small change in model parameters can lead to a loss of equilibrium. The dynamic model of the competitive market of three interchangeable goods is presented as a system of nonlinear ordinary differential equations with respect to the price vector. The model is built on the basis of accounting for the economic forces of sellers, buyers, the state and the economic forces of competition. The functional expressions of such forces are used, taking into account the economic laws of the commodity market. In the model, it is assumed that the volume of sales is a function of the price, which depends on the elasticity parameters, and the economic system is in equilibrium. The problem of equilibrium stability in the main and in the critical case of one zero root is studied. The conditions of asymptotic stability of the equilibrium are defined for the general case and for individual cases with different assumptions about the parameters of the model. The conditions of sustainable market development for symmetric and overlapping characteristics of competing goods are obtained. The results of the study showed that the stability of the market depends significantly on such factors as the elasticity of demand, the level of taxation and the intensity of competition. The figure shows regions of stability and instability of equilibrium depending on the values of the main parameters of the model. The obtained results can be used to develop recommendations for regulating the market and forecasting its behavior in various economic conditions.

Key words: *dynamic model, market of three goods, interchangeable goods, economic equilibrium, sustainability.*

Білоусова Т. П. Дослідження ринку трьох взаємозамінних товарів на стійкість
У цій роботі представлено дослідження динаміки ринку трьох взаємозамінних товарів на основі математичної моделі, що враховує взаємодію між продавцями, покупцями та державою. Аналізується стійкість рівноважного стану системи за різних значень параметрів моделі. Отримано аналітичні вирази для умов стійкості рівноваги. Досліджено вплив еластичності попиту та оподаткування на стійкість ринку. Показано, що за певних умов навіть невелика зміна параметрів моделі може призвести до втрати рівноваги.

Динамічна модель конкурентного ринку трьох взаємозамінних товарів представлена як система нелінійних звичайних диференціальних рівнянь щодо вектора цін. Модель будується на основі обліку економічних сил продавців, покупців, держави та економічних сил конкуренції. Використані функціональні вирази таких сил з урахуванням економічних законів товарного ринку. В моделі передбачається, що обсяг продажів є функцією ціни, яка залежить від параметрів еластичності, а економічна система має рівновагу. Досліджена задача про стійкість рівноваги в основному та в критичному випадку одного нульового кореня. Визначено умови асимптотичної стійкості рівноваги для загального випадку та для окремих випадків з різними припущеннями про параметри моделі. Отримано умови сталого розвитку ринку для симетричних та тих, що збігаються, характеристик товарів-конкурентів. Результати дослідження показали, що стійкість ринку істотно залежить від таких факторів, як еластичність попиту, рівень оподаткування та інтенсивність конкуренції. На малюнку представлені області стійкості та нестійкості рівноваги залежно від значень основних параметрів моделі. Отримані результати можуть бути використані для розробки рекомендацій щодо регулювання ринку та прогнозування його поведінки у різних економічних умовах. Дано економічне тлумачення результатів.

Ключові слова: *динамічна модель, ринок трьох товарів, взаємозамінні товари, економічна рівновага, стійкість.*

Introduction. The interest in mathematical models in economics among theoretical economists, as well as among specialists in applied mathematics, is growing every year. The study focuses, in particular, on the problems of existence and stability of equilibrium, for the solution of which effective mathematical methods are used. In the process of studying the properties of various economic phenomena, the development of methods for constructing economic models is not left without attention. The historical paths of development of mathematical methods in economics convincingly prove that the construction of models depends on the nature of accounting and the methods of reflecting events occurring in the economy.

In economic theory, first-order models, i.e. dynamic models that use only the first derivative $p(t)$ of the price of a good p as a function of time, have been studied since the time of the founders of mathematical methods of economics, L. Walras and A. Marshall [1-2]. Second-order models (using the second derivative $\ddot{p}(t)$), on the contrary, appeared much later and are not often encountered in the scientific literature [3]. The main advantage of second-order models is that they allow one to take into account information about price trends from both consumers and producers, and to study the influence of such information on the dynamics of market prices. The work [4] presents a first-order mathematical model of a competitive market of two interchangeable and complementary goods or services, where the sought-after functions of time are commodity prices. The conditions of stability of market equilibrium in the main and in a number of critical cases are studied and certain characteristics of an economic nature are identified. Since these models are described by a system of nonlinear differential equations, the analysis of such systems is associated with the well-known difficulties of the theory of differential equations. In works [5], special cases of a market for one and two goods are studied in detail, where, along with the study of the problem of the stability of economic equilibrium, the problem of optimal tax policy is formulated and solved, and other problems are also considered. As for the analysis of the dynamics of the market for three goods, the picture is different here; only individual works are known, based, as a rule, on the Walras model [6], where basically only one parameter is used – excess demand.

Formulation of the problem. This article examines a model of the market for three interchangeable goods or services [5-6], which takes into account the dependence of price behavior on the lower and upper values of the range of acceptable prices, the price elasticity of demand, and the level of influence of the main participants in trade – buyers, sellers, the state, and competitive forces.

Presenting main material. *1. Let us consider a mathematical model of the market [6] of three interchangeable competing goods:*

$$\dot{p}_j = -\frac{v_j p'_j\left(p_j - p_j^0\right)}{p_j - p_j^*} - \frac{d_j p''_j\left(p_j - p_j^0\right)}{p_j^{**} - p_j} + \frac{r_j}{q_j^0}\left(p_j q_j\left(p\right) - p_j^0 q_j^0\right) - \tag{1}$$

$$-\sum_{i=1, i\neq j}^{3} c_{ji}\left(\left(p_j - p_j^0\right) - \left(p_i - p_i^0\right)\right), \quad j = 1,2,3.$$

which is defined in the parallelepiped of admissible prices

$$p_j^* < p_j < p_j^{**}, j = 1,2,3, \tag{2}$$

where v_j, d_j, r_j and c_{ji} are constant coefficients reflecting the intensities of economic forces for sellers, buyers, the state and competitive forces, respectively; $p_j(t)$ is the price of one unit of the j-th good at time t; p_j^0 is the equilibrium price of the j-th good; $q_j(t)$ is the quantity of units of the j-th good sold at time t; q_j^0 is the equilibrium quantity of units of the j-th good; p_j^* – the lower threshold value of the price of the j-th product associated with the seller's costs; p_j^{**} – the upper (ceiling) value of the price of the j-th product, above which buyers refuse to purchase this product; p'_j – the seller's price surplus, $p'_j = p_j^0 - p_j^*$; p''_j is the buyers' surplus price, $p''_j = p_j^{**} - p_j^0$.

Let for the j-th competitor product $q_j = q_j(p)$ be the function of sales volumes of the price vector $p = (p_1, p_2, p_3)$ linear and given by the formula

$$q_j(p) = q_j^0 - e_j\frac{q_j^0}{p_j^0}\left(p_j - p_j^0\right) + \sum_{i=1, i\neq j}^{3} e_{ji}\frac{q_j^0}{p_i^0}\left(p_i - p_i^0\right), \tag{3}$$

where $e_j > 0$ and $e_{ji} > 0$ are the price elasticity of demand for the j-th good and the cross-elasticity of demand for the j-th good with respect to the price of the i-th good (at point $p = p^0$), respectively.

2. We study the stability of the economic equilibrium $p_j = p_j^0$, $j = 1,2,3$, of system (1) under conditions (2), (3) to the first approximation, i.e., for sufficiently small disturbances. To do this, we make a change of variables in system (1): $x_j = p_j - p_j^0$, $j = 1,2,3$.

Then, for the components of the vector $x = (x_1, x_2, x_3)$ such that $p'_j < x_j < p''_j$, we obtain a system of equations

$$\dot{x}_j = -\frac{v_j p'_j x_j}{x_j + p'_j} - \frac{d_j p''_j x_j}{p''_j - x_j} - c_j x_j + \sum_{i=1, i\neq j}^{3} c_{ji} x_i + \frac{r_j}{q_j^0}\left(\left(x_j + p_j^0\right) q_j\left(x + p^0\right) - p_j^0 q_j^0\right),$$

Where $c_j = \sum_{i=1, i\neq j}^{3} c_{ji}$, and the sales volume function

$$q_j\left(x + p^0\right) = q_j^0 - e_j\frac{q_j^0}{p_j^0}x_j + \sum_{i=1, i\neq j}^{3} e_{ji}\frac{q_j^0}{p_i^0}x_i.$$

After substituting the function $q_j\left(x + p^0\right)$ we obtain respectively

$$\dot{x}_j = -\frac{v_j p'_j x_j}{x_j + p'_j} - \frac{d_j p''_j x_j}{p''_j - x_j} + \left(-c_j + r_j\left(1 - e_j\right)\right)x_j + r_j\sum_{i=1, i\neq j}^{3}\left(c_{ji} + e_{ji}\frac{p_j^0}{p_i^0}\right)x_i -$$

$$-\frac{r_j}{p_j^0}e_j x_j^2 + r_j x_j\sum_{i=1, i\neq j}^{3}\frac{e_{ji}}{p_i^0}x_i, \tag{4}$$

$$-p'_j < x_j < p''_j, \quad j = 1,2,3.$$

The economic equilibrium is now the origin of coordinates $x_1 = x_2 = x_3 = 0$.

Let us select in system (4) a linear approximation in the vicinity of the point $x = 0$:

$$\ddot{x}_j = -S_j x_j + \sum_{i=1, i\neq j}^{3} C_{ji} x_i, \; j = 1,2,3. \quad (5)$$

where S_j is the safety margin of the market for the j-th product, $S_j = v_j + d_j - r_j\left(1 - e_j\right) + c_j$ and it is assumed that

$$C_{ji} = c_j + \left(r_j e_{ji} p_j^0\right) / p_i^0, \; i, j = 1,2,3 \; \left(i \neq j\right). \quad (6)$$

Equations (5) can be defined as a linear model of the three-goods market in variables $x_j = p_j - p_j^0$ – price deviations from equilibrium states. Let us highlight a number of individual cases.

Homogeneity of market characteristics of competing products. Let us assume that the goods being sold have a certain identity of some of their parameters in such a way that the matrix $C = \left(C_{ji}\right)$ of the coefficients of model (5) is symmetrical. Then, for simplicity, we can set $C_{12} = C_{21} = C_1$, $C_{23} = C_{32} = C_2$, $C_{13} = C_{31} = C_3$ and write the characteristic equation of the matrix of system (5), (6) in the form

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \quad (7)$$

$$a_1 = \left(S_1 + S_2 + S_3\right); \; a_2 = S_1S_2 + S_2S_3 + S_3S_1 - C_1^2 - C_2^2 - C_3^2;$$

$$a_3 = S_1S_2S_3 - C_2^2S_1 - C_3^2S_2 - C_1^2S_3 - 2C_1C_2C_3.$$

Therefore, the conditions of asymptotic stability of economic equilibrium (the Routh-Hurwitz criterion for (7)) are written in the form $a_1 > 0, \; a_3 > 0, \; a_1a_2 > a_3$.

In detailed notation they have the form of ratios

$$S_1 + S_2 + S_3 > 0; \; S_1S_2S_3 - S_1C_2^2 - S_2C_3^2 - S_3C_1^2 - 2C_1C_2C_3 > 0;$$

$$\left(S_1 + S_2 + S_3\right)\left(S_1S_2 + S_2S_3 + S_3S_1 - C_1^2 - C_2^2 - C_3^2\right) > S_1S_2S_3 - S_1C_2^2 - S_2C_3^2 - S_3C_1^2 - 2C_1C_2C_3. \quad (8)$$

It is quite difficult to give a complete economic analysis of inequalities (8). Let us highlight cases that correspond to certain market situations. Let us assume that the following conditions are met: the economic power of the state has the same effect on all sellers; the cross-price elasticities of demand for all goods are the same; the safety margins of two of the three competing products are the same.

For model (5) these conditions meet the following requirements:

$$r_j = r, \; e_{ji} = e, \; p_j^0 = p_i^0 = p^0, \; C_{ji} = C = c + re, \; i, j = 1,2,3, \text{ end } S_2 = S_3. \quad (9)$$

Taking into account (9), the asymptotic stability conditions (8) can be transformed to the form

$$S_1 + 2S_2 > 0, \; \left(S_2 + C\right)\left(S_1S_2 - CS_1 - 2C^2\right) > 0; \; \left(S_1 + S_2 - C\right)\left(S_1S_2 + S_2^2 + CS_2 - C^2\right). \quad (10)$$

We can investigate this system of inequalities with respect to the variable S_1, and then move to the plane.

Analysis of conditions (10) yields a solution on the plane of variables (S_1, S_2) in the form of a set of points, schematically represented by two shaded areas (Fig. 1). Note that, along with the region where all safety margins are positive (in the first quarter $S_1 > 0$, $S_2 = S_3 > 0$), there is also a region where either $S_1 < 0$, $S_2 = S_3 > 0$, or $S_1 > 0$, $S_2 = S_3 < 0$.

Absolutely symmetrical case. Let us now assume that the goods presented on the market are symmetrical in their characteristics, namely, along with conditions (10), the conditions of constancy of parameters are also satisfied

$$v_j = v,\ d_j = d,\ e_j = e,\ j = 1,2,3. \qquad (11)$$

Then the safety margins of the goods market are identical, $S_j = S$, and system (5) takes the form $\dot{x}_j = -Sx_j + C\sum_{i=1, i\neq j}^{3} x_i$, $j = 1,2,3.$

Where $S = v + d - r(1-e) + 2c$, $C = c + re^*$.

Here the characteristic equation is represented by the equality

$$(\lambda + S + C)^2 (\lambda + S - 2C) = 0. \qquad (12)$$

Therefore, if $S > 2C$, then all roots have negative real parts, which corresponds to the asymptotic stability of the equilibrium.

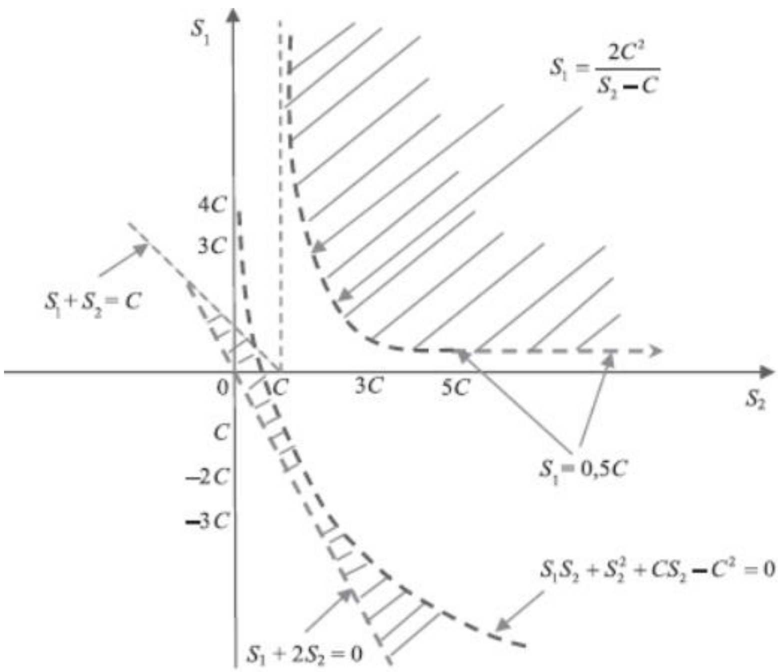

Fig. 1. Region of asymptotic stability

For $S < 2C$, the equilibrium of system (5) is unstable. Let us consider the case of $S > 2C$. Given the accepted notations, this is equivalent to the inequality

$$r(1 - e + 2e^*) < v + d. \qquad (13)$$

Depending on the sign of the quantity $(1 - e + 2e^*)$, two cases should be analyzed.

1. If $e < 1 + 2e^*$ (limited price elasticity of demand), then the equilibrium is asymptotically stable when condition (13) is met. It follows from this that in the case of limited price elasticity of demand, an increase in the tax rate may lead to a disruption of market stability.

2. If the reverse inequality is satisfied $e \geq 1+2e^{*}$ *(elastic demand for price),* then condition (13) is always satisfied. Thus, a market of three homogeneous goods with elastic price demand is able to function stably under high taxes.

Similarly, we obtain the instability of the economic equilibrium of model (1) if $e<1+2e^{*}$, $r\left(1-e+2e^{*}\right)>v+d$, which, in conditions of limited price elasticity of demand, means a fairly high level of taxation.

Critical case of one zero root. Let us study the problem of stability of a nonlinear model in an absolutely symmetric case (fulfillment of equalities (9) and (11)), when one of the roots of the characteristic equation (12) is equal to zero. The following possible situations follow from equality (12).

At $S=2C$ we have a critical case of one zero root. Here the other two roots $\lambda_1=-3C$ and $\lambda_2=-6C$ are negative and the problem of equilibrium stability requires additional research. When $S=-C$ we have two zero roots and one positive root $\lambda=3C$, which means that the equilibrium is unstable. The critical case of having a pair of purely imaginary roots is impossible. Thus, in the nonlinear model (4) the situation when $S=2C$, i.e. $v+d=r\left(1-e+2e^{*}\right)$, remains unexplored. Let it be done. Let us write the model system (4) in the absolutely symmetric case, representing its right-hand sides using Taylor series in the vicinity of the origin R^3 with an accuracy of up to terms of the third order of smallness. We obtain a system of equations

$$\begin{aligned}
\dot{x}_1 &= -2Cx_1+Cx_2+Cx_3+H_2\left(x_1\right)^2-H_3\left(x_1\right)^3+Rx_1x_2+Rx_1x_3+o\left(\|x\|^3\right),\\
\dot{x}_2 &= Cx_1-2Cx_2+Cx_3+H_2\left(x_2\right)^2-H_3\left(x_2\right)^3+Rx_1x_2+Rx_2x_3+o\left(\|x\|^3\right),\\
\dot{x}_3 &= Cx_1+Cx_2-2Cx_3+H_2\left(x_3\right)^2-H_3\left(x_3\right)^3+Rx_3x_2+Rx_1x_3+o\left(\|x\|^3\right),
\end{aligned} \tag{14}$$

where

$$C=c+re^{*},\ H_2=\frac{v}{p'}-\frac{d}{p''}-\frac{re}{p^0},\ H_3=\frac{v}{\left(p'\right)^2}+\frac{d}{\left(p''\right)^2},\ R=\frac{re^{*}}{p^0}. \tag{15}$$

Substitution of variables $y_1=x_1+x_{2_3}+x_3$, $y_2=x_2$, $y_3=x_3$; $x_1=y_1-y_2-y_3$, $x_2=y_2$, $x_3=y_3$ reduces the linear approximation matrix of system (14) to the form $\begin{pmatrix}0&0&0\\C&-3C&0\\C&0&-3C\end{pmatrix}$. Let us write out the equations of the model in new variables with an accuracy of up to terms of the third order of smallness in the vicinity of the origin:

$$\begin{aligned}
\dot{y}_1 &= y_1^2H_2+2y_1y_2\left(R-H_2\right)+2y_1y_3\left(R-H_2\right)+2y_2y_3\left(H_2-R\right)+2y_2^2\left(H_2-R\right)+\\
&+2y_3^2\left(H_2-R\right)+H_3\left(-y_1^3+3y_1^2y_2-3y_1y_2^2+3y_2^2y_3+3y_2y_3^2+3y_1^2y_3-3y_1y_3^2\right),\\
\dot{y}_2 &= Cy_1-3Cy_2+y_2^2\left(H_2-R\right)+Ry_1y_2-H_3y_2^3,\\
\dot{y}_3 &= Cy_1-3Cy_3+y_3^2\left(H_2-R\right)+Ry_1y_3-H_3y_3^3.
\end{aligned} \tag{16}$$

According to the general theory of critical case studies [7], we find the solution $y_2=u_2\left(y_1\right)$ and $y_3=u_3\left(y_1\right)$ of the system of equations in implicit functions

$$Cy_1-3Cy_2+y_2^2\left(H_2-R\right)+Ry_1y_2-H_3y_2^3+\ldots=0,$$

$$Cy_1 - 3Cy_3 + y_3^2\left(H_2 - R\right) + Ry_1y_3 - H_3y_3^3 + \ldots = 0$$

in the form of rows $u_2\left(y_1\right) = \alpha_1 y_1 + \alpha_2 y_1^2 + \alpha_3 y_1^3 + \ldots$ and $u_3\left(y_1\right) = \beta_1 y_1 + \beta_2 y_1^2 + \beta_3 y_1^3 + \ldots$ with uncertain coefficients α_j and β_j. The standard procedure for finding the coefficients gives

$$\alpha_1 = \beta_1 = \frac{1}{3};\ \alpha_2 = \beta_2 = \frac{H_2 + 2R}{27C};\ \alpha_3 = \beta_3 = \frac{2\left(H_2 - R\right)\alpha_1\alpha_2 + R\alpha_2 - H_3\left(\alpha_1\right)^3}{3C}.$$

Thus, to within terms of the third order of smallness, we have

$$u_2\left(y_1\right) = u_3\left(y_1\right) = \frac{1}{3}y_1 + \frac{H_2 + 2R}{27C}y_1^2 + \frac{2\left(H_2 - R\right)\alpha_1\alpha_2 + R\alpha_2 - H_3\left(\alpha_1\right)^3}{3C}y_1^3. \qquad (17)$$

We now substitute expressions (17) into the first (critical) equation of the system. To do this, we first transform the second-order polynomials in the right-hand side of this equation (up to terms of the third order of smallness). We have

$$y_1^2H_2 + 2y_1y_2\left(R - H_2\right) + 2y_1y_3\left(R - H_2\right) + 2y_2y_3\left(H_2 - R\right) + 2y_2^2\left(H_2 - R\right) + 2y_3^2\left(H_2 - R\right) =$$

$$= \frac{H_2 + 2R}{3}y_1^2 + \frac{8R\left(H_2 + 2R\right)}{3^5C}y_1^3 + \ldots.$$

Proceeding in a similar manner with the third-order polynomials of the first equation (16), we obtain the representation

$$H_3\left(-y_1^3 + 3y_1^2y_2 - 3y_1y_2^2 + 3y_2^2y_3 + 3y_2y_3^2 + 3y_1^2y_3 - 3y_1y_3^2\right) = H_3\frac{5}{9}y_1^3 + o\left(\left|y_1\right|^3\right).$$

Thus, the critical equation is transformed to the form

$$\dot{y}_1 = \frac{H_2 + 2R}{3}y_1^2 + \frac{135CH_3 + 8R\left(H_2 + 2R\right)}{3^5C}y_1^3 + o\left(\left|y_1\right|^3\right), \qquad (18)$$

where the values of H_2, H_3 and R are determined by formulas (15). Taking into account the results of the study of the critical case of one zero root [7], we find that at $H_2 + 2R \neq 0$ the equilibrium is unstable. If $H_2 + 2R = 0$, then the coefficient of y_1^3 in expression (18) is positive, and we also have a case of equilibrium instability. The results of the research of the absolutely symmetric case will be presented in the form of a statement.

Statement: *Suppose that conditions (9) and (11) are satisfied for model (1). Then the economic equilibrium* $p_j = p_j^0,\ j = 1,2,3$*, will be asymptotically stable in each of the following cases:*

1) $e \geq 1 + 2e^*$
2) $e < 1 + 2e^*$, $v + d > r\left(1 - e + 2e^*\right)$.

The equilibrium will be unstable if:

3) $e < 1 + 2e^*$, $v + d \leq r\left(1 - e + 2e^*\right)$.

Here H_2, H_3, C, R are determined by formulas (15).

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